Financial Innovation, Macro-prudential Policies and Leverage Cycles

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Abstract

In the wake of the 2008 global financial crisis, the Federal Reserve has initiated various macro-prudential policies to stabilize asset prices and stimulate the real economy. These policies achieved the targets during the recession in 2011, but failed in this task during the recovery in 2014. This paper rationalizes these observations on the effectiveness of macro-prudential policies in a three-period general-equilibrium model with incomplete markets, heterogeneous agents and endogenous leverage. The crucial insight is the interaction between belief heterogeneity and financial innovation at equilibrium. Financial innovation, as an additional macro-prudential tool, alongside collateral requirements on borrowing, takes the form of collateral protection insurance (CPI) contracts. The results show that macro-prudential policies which stabilize asset prices, may not improve social welfare. Specifically, policies that regulate higher collateral requirements stabilize asset prices, smooth the leverage cycle, but reduce social welfare, when belief heterogeneity is small. In contrast, the introduction of CPI contracts increases the dispersion of asset prices across date events, exacerbating the leverage cycle, but improves social welfare, when belief heterogeneity is large.

1 Introduction

After the 2008 financial crisis, economies have suffered from depressed asset prices and low aggregate consumption. Also, it has been observed that high leverage volatility was followed by large fluctuations of asset prices. Historically, there were leverage cycles: higher leverage was followed by higher asset prices, and lower leverage was followed by lower asset prices. A vast literature (Korinek & Simsek, 2016; Rubio & Carrasco-Gallego, 2014) proposed macro-prudential tools like loan-to-value caps and leverage restriction targeting asset price stability and welfare improvement. These instruments have been implemented by U.S supervisors. During the recession in 2011, the policy targets were achieved. However, when the US economy recovered in 2014, the application of macro-prudential instruments was followed by more volatile asset prices and fewer consumption expenditures, which implies lower social welfare.

This paper explains the effects of macro-prudential policies on asset prices and social welfare in a three-period general-equilibrium model with incomplete markets, heterogeneous agents and collateral. The crucial insight is the interaction between belief heterogeneity and financial innovation at equilibrium. I show that macro-prudential policies which stabilize asset prices may not induce welfare improvement.

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Higher collateral requirements which smooth the leverage cycle reduce social welfare when belief heterogeneity is small. Moreover, this paper introduces financial innovation as an additional macro-prudential policy, alongside higher collateral requirements on borrowing. The financial innovation in the form of collateral protection insurance (CPI) contracts, assists financial institutions to mitigate the catastrophic losses stemming from severe financial crises. CPI is known as one type of vehicle insurance. If households borrow for buying a car, they are required to buy CPI. Insurance companies as CPI sellers will repay lenders once borrowers default. I show that the introduction of CPI contracts exacerbates the leverage cycle but improves social welfare when belief heterogeneity is large.

By incorporating collateral and agent heterogeneity, the model interprets the four facts I explain below. I use the standard derivation of the Production Price Index (PPI) in the industry of industrial machinery manufacturing to measure the volatility of asset prices. For example, the volatility of PPI in 2011.I is the standard derivation of monthly PPI from 2010-03-01 to 2011-03-01. The industrial machinery is a subsector of the industry that produces and maintains machines for consumers and other firms. As it requires initial investment and generates a long-term return, it fits the asset described in the model. Moreover, this paper uses the real personal consumption expenditures on non-durable goods to index social welfare.

Stylized fact 1: *Fluctuations of asset-backed commercial paper (ABCP) spread during the global financial crisis.*

Figure 1 displays trades of asset-backed commercial paper (ABCP), which facilitates the flow of funds between entrepreneurs and investors. From 2007-2009, there was high volatility of ABCP spread from 2007-2009. It went up from the first season of 2007. From this time point, ABCP sellers should offer higher returns.

Stylized fact 2: *After Comprehensive Capital Analysis and Review (CCAR) stress tests were applied, the volatility of asset prices decreased, and real personal consumption expenditures increased.*

In the third quarter of 2011, CCAR stress tests were adopted. CCAR is an annual exercise by the Federal Reserve to assess whether the largest bank holding companies operating in the US have sufficient capital to continue operations throughout the financial stress.

The volatility of the Producer Price Index (PPI) in the industry of industrial machinery manufacturing dropped greatly in the third quarter of 2011. Also, the real personal consumption expenditures on non-durable goods increased between the third quarter of 2011 and the second quarter of 2012.

Stylized fact 3: *After Inter-agency Guidance on Leverage Lending (IGLL) took effect, the volatility of asset prices increased, and real personal consumption expenditures decreased.*

In the first quarter in 2013, the Inter-agency Guidance on Leverage Lending (IGLL) was published and in the last quarter of 2014 Frequently Asked Questions (FAQ) notice was issued, to address strong growth in leveraged lending after the 2008 financial crisis. The IGLL and FAQs focus on the size and risk profile of the leveraged activities of an institution, relative to its assets, capital, liquidity and earnings.

Calem, Correa, and Lee (2020) found that the IGLL had a significant effect on loan trades only after the FAQs were published. Figure 2 shows that in the last quarter of 2014, the volatility of PPI increased and the real personal consumption expenditures on non-durable goods decreased.

Stylized fact 4: *After the G-SIB capital surcharges was applied, the volatility of asset prices increased, and real personal consumption expenditures increased.*

In the second quarter of 2014, each global systemically important bank holding company (G-SIB) will have to finance its assets with additional capital to increase its resiliency in light of the greater threat they pose to the financial stability in the US. One goal of G-SIB capital surcharges is to make government bailouts of G-SIBs less likely by having G-SIBs self-insure themselves against severe financial crises. This policy took effective in the first quarter of 2016. Figure 2 shows that in the first quarter of 2016, the volatility of PPI increased while real personal consumption expenditures on non-durable goods increased.

Fact 1 suggests that heterogeneous beliefs on collateral value between lenders and borrowers could be reflected by the ABCP trades.[1] Figure 1 illustrates that the significant increase of ABCP spread in 2007 is before the bankruptcy of Lehman Brothers which happened in the fourth season of 2008. This could be attributed to the lenders' perceived reduction of the collateral value. Thus, a high ABCP spread could imply large belief heterogeneity.

Facts 2 and 3 suggest that belief heterogeneity affects the effectiveness of macro-prudential policies. Figure 2 shows that the belief heterogeneity was large when CCAR stress tests were issued. This moment is after US stock markets crashed in the second quarter of 2011[2]. By contrast, the belief heterogeneity was small when IGLL took effect. Since the second quarter of 2014, the US economy began to recover and GDP rebounded strongly. This implies that macro-prudential policies may increase social welfare and reduce the volatilities of asset prices during the recession. However, they may reduce social welfare and increase the volatilities of asset prices during the recovery. Lastly, Fact 4 suggests that the requirement on enhancing resiliency of financial institutions may increase the volatilities of asset prices, but still improve social welfare.

This paper develops a framework that rationalizes these patterns in the stylized facts driven by the macro-prudential policies. In particular, CPI is introduced as a new type of collateral to interpret the patterns in Fact 4. Since CPI as an additional collateral assists financial institution to self-insure against severe crises, CPI is a relevant financial instrument to lower the lenders' loss of G-SIB default. Although a myriad of other factors could have contributed to the aforementioned facts during this time period, this paper ignores them.

As previously noted, collateral and belief heterogeneity play important roles in the model. I present a three-period, general-equilibrium model along the lines of Geanakoplos (2010a) model of the leverage cycle. The core element of the analysis is the repayment enforce-ability problem: Suppose that if borrowers default, they cannot be forced to repay except by seizing collateral. Agents should post collateral in order to issue promises. The collateral prices determine both the funding to borrowers and the repayment to lenders. The economy has three periods with uncertainty about the state of nature. The capital investment pays out in the last period, and in the middle period information arrives about the likelihood of the final payoffs. If there is good news in the middle period, uncertainty about the capital return decreases, otherwise, the uncertainty increases. There are three types of agents that hold heterogeneous beliefs about the realization of the recession: optimists, moderates and pessimists. Agents trade loans to borrow and lend. Optimists have access to a technology that allows them to invest in capital in the initial period, while moderates and pessimists are constrained by technology barriers and buy capital goods from optimists in the middle period.

I analyze two economies. The first one is defined as Leverage-economy or L-economy. Leverage describes that capital goods as collateral support borrowers to issue loans. Across states, equilibrium leverage is not constant. Borrowing capacities fluctuate with endogenous leverage. The variations of leverage drive the business cycle. In the second, CPI is introduced into the previous leverage economy. I call this CPI-economy. CPI contracts as an additional type of collateral backing up loans affect

[1]The collateral of ABCP includes assets under repos, trade receivables, structured finance securities, commercial loans, residential mortgages. Although this paper focus on the collateral held by agents with productive technology, ABCP is a good example of short-term collateralized lending market.

[2]In August 2011, there was a sharp drop in stock prices across the US. Because of the slow economic growth of the US, the US sovereign debt was downgraded.

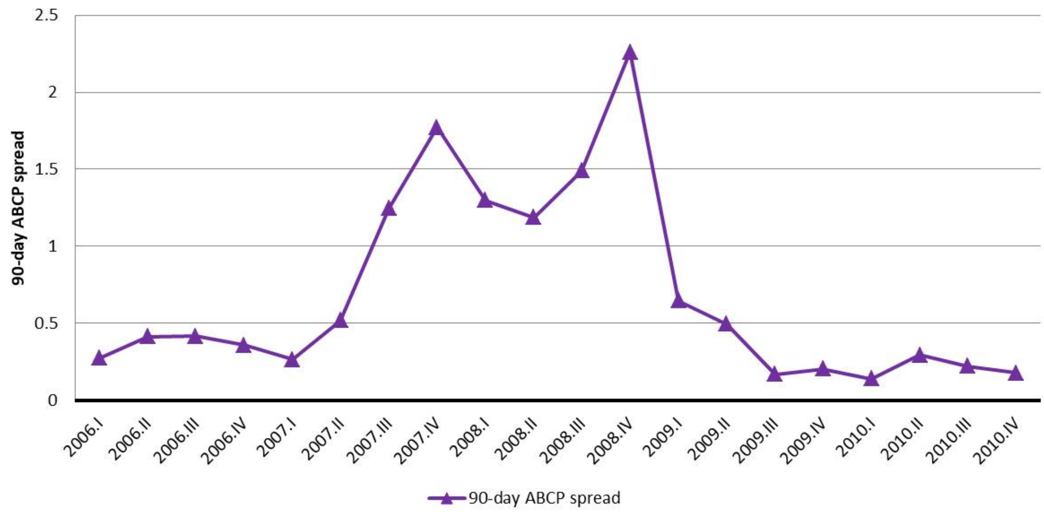

Figure 1: Collateralized lending in US

Notes: Figure 1 shows the trades of asset-backed commercial paper (ABCP). The ABCP spread measures the difference between the return of the ABCP and the return of a treasury bill with similar maturity.

Source: Federal Reserve Bank-Business Finance Data.

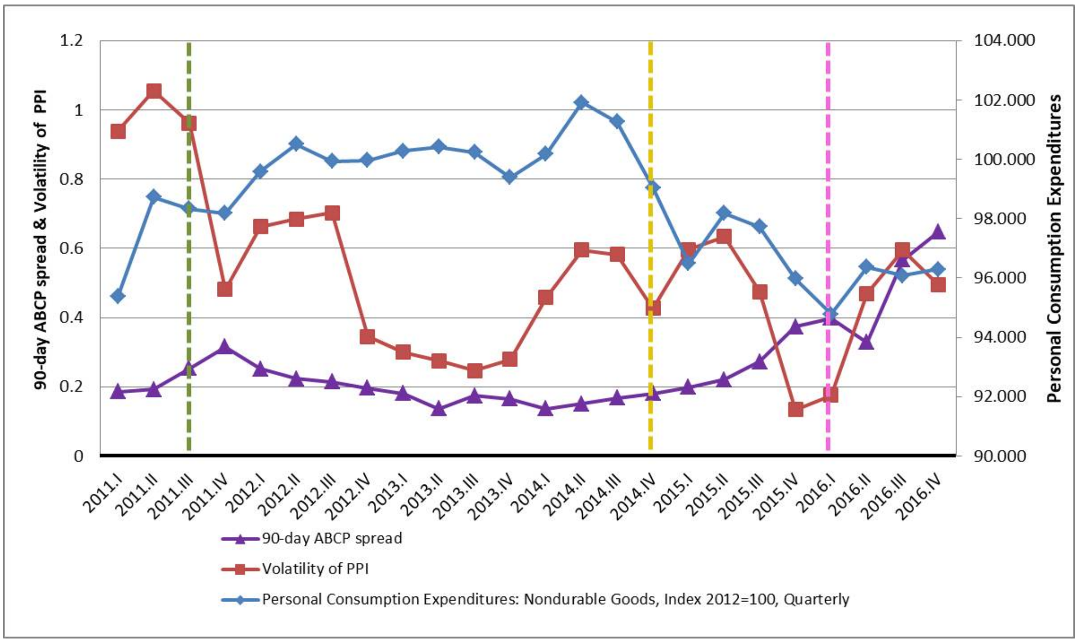

Figure 2: Application of macro-prudential policies in US

Notes: The purple line shows the trades of asset-backed commercial paper (ABCP). The ABCP spread measures the difference between the return of the ABCP and the return of a treasury bill with similar maturity. The red line shows the volatility of Producer Price Index (PPI) in the industry of Industrial Machinery Manufacturing from 2011 to 2015. The volatility is measured by the standard derivation of the PPI in one year. For example, the volatility of PPI in 2011.I is the standard derivation of monthly PPI from 2010-03-01 to 2011-03-01. The blue line shows how much agents spent on non-durable goods from 2011 to 2015, based on the price level in 2012. The green dashed line marks the time point when CCAR stress tests took effect. The yellow dashed line marks the time point when IGLL took effect. The pink dashed line marks the time point when higher collateral requirement on G-SIB took effect.

Source: Federal Reserve Bank-Business Finance Data.

endogenous leverage and thus influence the leverage cycle. CPI trades are only between optimists and moderates. Because pessimists always “ask” higher prices for selling CPI relative to moderates, optimists will always go for cheaper CPI contracts in equilibrium.

I present a series of numerical simulations to rationalize the patterns in the stylized facts. The ABCP spread is modeled as the promised return over the loan price. The model simulations show that the loan premium is an increasing function of belief heterogeneity. In other words, lenders expect higher compensation from borrowers when they believe that default is very likely. Additionally, macro-prudential policies, CCAR stress tests and IGLL are linked with leverage. Acharya, Engle, and Pierret (2014) used the leverage ratio to measure the effect of stress tests and Kim, Plosser, and Santos (2018) showed that IGLL was effective at reducing the banks’ leveraged lending activity. Since leverage is a decrease function of collateral requirements, this paper models CCAR stree tests and IGLL by increasing collateral requirements on borrowing. The simulation results show that when belief heterogeneity is large, increasing collateral requirements on borrowing reduces the volatility of asset prices and improves social welfare. However, when belief heterogeneity is small, this macro-prudential instrument reduces the volatility of asset prices but reduces social welfare. Moreover, the G-SIB capital surcharges induce a high cost on banks to comply with the requirement. The need for cost reduction may lead to financial innovation (Vousinas, 2015). As additional capital is required to absorb the potential losses of risky assets, this macro-prudential tool is modeled as requiring borrowers to hold CPI contracts. The simulation of the CPI-economy illustrates that if financial institutions are required to use CPI to self-insure, the volatility of asset prices and social welfare increases.

The crucial insight is that belief heterogeneity influences the cost of borrowing and the repayments to lenders. If agents share similar beliefs on collateral value, the loan premium is low and asset prices fall by little in the recession. This translates to low borrowing cost and higher repayment rates to lenders even if borrowers default. Even though increasing collateral requirements on borrowing reduces the losses buyers face in recessions, it increases the cost of borrowing and thus reduces productive savings. Thus, since the macro-prudential policy increase the borrowing cost by large, the beneficial impact of the macro-prudential policy on lenders cannot offset the harm on borrowers. The social welfare declines. On the other hand, if there is an extreme belief disagreement between borrowers and lenders, lenders suffer from a severe loss induced by a large decline in asset prices. Since the macro-prudential policy increases the repayments to lenders by large, the benefit compensates for the expense induced by a higher borrowing cost. The social welfare increases. Moreover, requiring financial institutions to self-insure also increases the repayment to lenders. As the CPI contract is cheaper collateral compared with assets, the financial innovation reduces the borrowing cost of financial institutions. As a result, the social welfare increases.

I show that the macro-prudential perturbations have three types of effects on social welfare. First, heterogeneity effects arise because there are differences in the marginal rates of substitution across states of the world among agents. Second, collateral effects arise because collateral regulation has direct effects on agents’ borrowing constraints. Third, pecuniary externalities arise because collateral regulations change asset prices. Therefore, the effectiveness of macro-prudential policies is determined by the sign and magnitude of the aforementioned effects. I identify sufficient conditions that characterize unambiguously the impact of the macro-prudential policies on social welfare. These conditions take the form of the difference in beliefs of agents, the net trading positions of loans and assets, the asset prices in the recession, the cost of satisfying collateral constraints and the sensitivity of asset prices to the perturbations.

The simulation results also illustrate that increasing collateral requirements on borrowing lowers welfare through tighter collateral constraints; on the other hand, tightening collateral regulations increases welfare through effects arising from heterogeneous beliefs and stabilized asset prices. By contrast, introducing CPI contracts lowers social welfare due to unstable asset prices but improves social welfare considering heterogeneous beliefs. Also, a tighter collateral requirement on CPI contracts has no effect on social welfare. The result also shows that heterogeneity effects and collateral effects are always dominant, while pecuniary externalities are negligible. It means that policy authorities should focus on motivating productive investment rather than stabilizing asset prices if lenders and borrowers share similar beliefs on investment projects. Additionally, CPI contracts as an additional type of collateral facilitate borrowers to enhance their borrowing capacity without lowering their resiliency. Therefore, this macro-prudential tool induces welfare improvement though it destabilizes asset prices.

1.1 Related literature

This paper is closely related to the work of Geanakoplos (1997) who pioneered the general equilibrium analysis of collateralized lending. Geanakoplos (2003) introduced the idea of equilibrium leverage. He also identified an increase in belief heterogeneity as a cause of decreased leverage and hence of the leverage cycle. The leverage cycle, defined by Geanakoplos (2010a), means that there are times when leverage is high, agents can invest with little money as the down-payment, and times when leverage is low, agents should have enough money in hand to invest. The baseline setting captures no default with riskless loan contracts. Geanakoplos (2010b) suggested the leverage should be monitored and limited in the ebullient times to avoid a plunge of capital prices induced by an out-sized leverage cycle. In addition, Araujo, Kubler, and Schommer (2012) and Simsek (2013) follow the model of Geanakoplos (1997). In their model, however, the loans traded in the equilibrium are risky, as they focused on the negative effect of default on the real economy. Araujo et al. (2012) show how regulations on leverage, that is, restricting the sets of tradable assets through intervention on collateral requirements, affect social welfare. These arguments motivate this research to investigate how collateral regulations influence the leverage cycle if borrowers' default can only be eliminated by tighter collateral requirements.

Another strand of the theoretical literature concerns the effect of borrowing constraints on asset prices, for example, Kiyotaki and Moore (1997), **?**, Brunnermeier and Pedersen (2009), Fostel and Geanakoplos (2012), Brunnermeier and Sannikov (2014). Borrowing decisions made in the good times of the business cycle destabilize collateral prices, especially when borrowers default in the recession. Stabilizing asset prices across the business cycle calls for macro-prudential policies that limit excessive credit growth and leverage (Rubio & Carrasco-Gallego, 2014). These prevention policies take the form of tax on debt, relying on the argument that over-borrowing during normal, non-crisis times affect collateral prices in the downturn (Bianchi & Mendoza, 2018). However, Benigno, Chen, Otrok, Rebucci, and Young (2013) pointed out that adopting the macro-prudential policies may be costly in welfare terms. If a small tax on debt eliminates the probability of a crisis, the decrease of average consumption reduces social welfare. This paper shows how the macro-prudential perturbations on borrowing with collateral influence asset prices and social welfare.

There is a strand of empirical work which shows that agents actually form different beliefs and how heterogeneous beliefs influence asset prices. Frankel and Froot (1985) did a survey which explains the observed volatility of foreign exchange rates by belief heterogeneity. Moreover, Kandel and Pearson (1995) conduct an empirical research of heterogeneous beliefs of investors. They showed that the stock returns are influenced by different interpretations of public information by investors. This paper is also related to the literature concerning the plausibility of belief heterogeneity in collateralized lending. Simsek (2013) stated that the borrowing constraint stems from the belief heterogeneity between borrowers and lenders. Optimists borrow from pessimists to invest in projects. Pessimists might be reluctant to lend because they do not value optimists' collateral (the project) as much as optimists do (Simsek, 2013). The level of belief heterogeneity influences the tightness of a constraint on optimists' ability to borrow. Geanakoplos and Zame (2014) also showed that the increase in belief heterogeneity decreases leverage and collateral prices, and thus increases borrowers' funding cost.

In addition, belief heterogeneity is plausible in analyzing effectiveness of policies regarding of social welfare. The seminal contributions of Kurz, Jin, and Motolese (2005), Branch and McGough (2009) and De Grauwe (2011) argued that the belief heterogeneity affects the efficacy of monetary policy. Wei (2020) proposes that the level of belief heterogeneity influences the effectiveness of setting loan-to-value caps. Brunnermeier (2014) proposed a belief-neutral welfare criterion that requires policymakers to be sure of the presence of belief disagreements among agents instead of taking a stand on whose belief is correct. These arguments motivate this research on how belief heterogeneity influences the effects of collateral regulations on social welfare.

Finally, this paper is related to the literature about the effects of financial innovation on the business cycle. Dynan, Elmendorf, and Sichel (2006) proposed that financial innovation such as developments in lending practices and loan markets enhances the ability of firms and households to borrow. In addition, Morrison (2005) stated that financial innovation like credit derivatives enables entrepreneurs to obtain cheap bond market finance, but social welfare decreases. The reason might be an under-pricing of the risk in a new financial environment, resulting in a surge in credit and asset prices (Boz & Mendoza, 2014). Morrison (2005) suggested this negative effect can be countered by the introduction of reporting requirements for credit derivatives. On the other hand, if financial innovation complements policy restrictions on borrowing constraints, economic activities are motivated. Thus, it motivates my research on how introducing financial innovation together with prudential regulations affects the volatility of asset prices and social welfare. Moreover, Fostel and Geanakoplos (2012) showed that financial innovation modeled as a structural change could generate another cycle characterized by higher endogenous leverage. This paper characterizes the leverage cycle with CPI contracts introduced.

Outline The remained is organized as follows. Section 2 presents the model environment. Section 3 presents agents' behavior in the equilibrium. The numerical examples show that under certain conditions, the above stylized facts emerge naturally in the equilibrium. Section 4 examines the mechanism and characterizes the conditions for welfare-improving. Section 5 presents robustness results. Section 6 concludes. Appendices A-C present supplemental material.

2 The Model

2.1 Basic Environment

The economy lasts for three periods, $t = 0, 1, 2$. The uncertainty is characterized by a tree of states $s \in S$ and $S = \{0, H, L, HL, HH, LL, LH\}$. H stands for the boom state, and L stands for the bust state. This paper denotes the time of s by $t(s)$, so $t(0) = 0$, $t(s_1) = 1$, and $t(s_2) = 2$. $s_1 \in S_1$, $S_1 = \{H, L\}$, and $s_2 \in S_2$, $S_2 = \{HH, HL, LH, LL\}$. Agents i's endowment of the consumption good is represented by e^i_s in each state $s \in S$. The endowments e^i_0 enable agents to lend or borrow. At date 0, there is one consumption good. The shock to the economy is uncertain. Agents transfer their wealth intertemporally by investing either financial contracts (q) or capital (k). At date 0, capital investment transfers α units of consumption goods into one unit of capital goods. At date 1, the uncertainty is realized by either the endowments or capital price, p_s. At date 2, the uncertainty is realized by the dividends of capital investment. Also, the capital dividends has the characteristic that good news reduces the uncertainty while bad news increases uncertainty. I assume, as shown in Figure 5, that after good news at $s = H$, the capital return equals $r^k_{HH} = r^k_{HL}$. However, after bad news at state $s = L$, I assume that $r^k_{LH} > r^k_{LL}$. This assumption is made for simplicity as this paper focuses on the recession which is induced by the combination of bad news.

Each agent i is characterized by a utility u^i, a discount factor β and different subjective probabilities $\pi^i_s, s \in S \setminus 0$. I assume that agents are risk-averse. Thus, in each state $s \in S$, the utility function is monotonic, differentiable and strictly concave. The expected utility to agent i is:

$$u^i = u^i_0(c^i_0) + \sum_{s \in S \setminus 0} \beta^{t(s)} \pi^i_s u^i_s(c^i_s). \tag{1}$$

Suppose agents have identical wealth, utilities and discount rates, but differ in their beliefs. This paper assumes that the beliefs are common knowledge among agents. Before discussing welfare properties in the equilibrium, it is useful to sort out different sources of belief agreements. Suppose the correct possibility π_s of the future states is unobservable. Three types of agents, optimists, moderates and pessimists are denoted by $i = 1, i = 2$ and $i = 3$ respectively. They observe some information about π_s and know each others' beliefs at date 0. This means three types of agents agree to disagree. The subjective possibilities π^i_s denotes the probability of reaching state s from its predecessor s_-, $s_- \in \{0, H, L\}$. This paper denotes Π to be the given distribution of the heterogeneous beliefs on the downturn, which is discrete and positive. Let σ denote standard derivation corresponding to the beliefs of three types of agents. A larger σ means agents have more distinctive belief on the bust state from each other. I suppose beliefs are captured by persistent individual heterogeneity, so each agent's belief is independent and identically distributed.

2.2 Financial contracts

The core part of this research includes financial contracts and collateral. The definition of financial contracts is an agreement including a promise and the collateral backing it and the definition of financial innovation is new types of promises backed by collateral, or the use of new types of collateral. There is no doubt that the payment of collateral depends on the future state of nature. As the collateral dividends are independent from the promise size and other decisions of contract sellers, the consideration of hidden effort is eliminated. Also, I assume that the collateral used to secure some contracts cannot back other

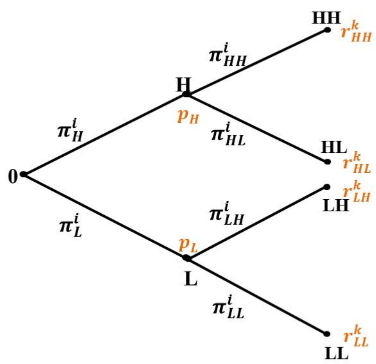

Figure 3: Capital return

contracts.

The price of financial contract ($q \in Q$) is m_s^q and the amount of contracts traded is θ_{sq}^i. A positive θ_{sq}^i indicates that agents i buy contracts q or saving $\theta_{sq}^i m_s^q$, while a negative θ_{sq}^i indicates that agents i sell contracts q or raise $\mid \theta_{sq}^i \mid m_s^q$ for financing their investment.

A one-period loan contract ($q = b \in Q^b$) promises 1 unit of consumption goods, backed by a amount of collateral $\boldsymbol{x}_s^b$ which has to satisfy the collateral requirement $\boldsymbol{C}_s^b \geq 0$. Thus, loan contracts ($Q^b$) are characterized by many alternative collateral requirements with the same promise. According to assumptions in the previous subsection, agents 1 only borrow in the state 0 and L. I assume that the collateral should be held by agents who take the short position of financial contracts. Since the return function of collateral required in the contracts is the same for each agent $\boldsymbol{F}_s^i(\boldsymbol{C}^b) = \boldsymbol{F}_s^{i'}(\boldsymbol{C}^b), \forall s \in S \setminus 0$, there is no adverse selection problem. Since the contract sellers lose the return of their collateral if they do not honor their promise, the delivery of loan contracts (r_s^b) at date 1 is

$$min\{1, \boldsymbol{F}_s^i(\boldsymbol{C}^b)\}.$$

The loan contract is priced at m_s^b and the traded amount is θ_{sb}^i. Each loan contract b has a distinct collateral requirement $\boldsymbol{C}_s^b = (\phi_s), s \in \{0, L\}$, where $\phi_s, s \in \{0, L\}$ is the collateral requirement on capital. Default at date 1 depends on capital prices p_s. If the capital price in the equilibrium satisfy the assumption, $\bar{p_L} < 1/\bar{\phi_0}$, $\bar{p_H} \geq 1/\bar{\phi_0}$, loan contract sellers only default in the state L.

The CPI contracts ($q = j \in Q^j$) is a derivative contract and its payoff depend on the actual payoff of loan contracts. As a new type of collateral, it provides additional collateral dividends for loan contracts. In the economy with CPI contracts, $\boldsymbol{C}_0^b = (\phi_0, h_0)$, where h_0 is the collateral requirement on CPI contracts at date 0. The collateral $\boldsymbol{x}^b = (k, \theta_{0j})$. I assume that the return of CPIs r_s^j is $(r_H^j, r_L^j) \equiv (0, 1 - \phi_0 \bar{p_L})$. The CPI contract is priced at m_0^j, and the traded amount is θ_{0j}^i. The cost of selling one unit of CPI is ε. Thus, the cost function is $T(\theta_{0j}^i) = 1_{\{\theta_{0j}^i < 0\}} \varepsilon \theta_{0j}^i$.[3] Only capital investors can buy CPI contracts.

Although I have supposed that the set of loan contracts that may be issued and the collateral requirement associated with each of them is given; the only contract actively traded has a endogenously determined collateral requirement. Suppose there are interventions requiring more collateral than the endogenous level. The actively traded loan contract has the collateral requirement exogenously determined by the interventions. I assume parameters are consistent with default in the bust state. The loans traded in the equilibrium are risky, so this model is particularly suitable to study economic issues associated with default of collateralized loans.

In the economy where only capital can be used as collateral to issue promise, $\boldsymbol{x}^b = (k)$. The intervention to reduce lenders' loss is increasing ϕ. In the L-economy, only capital can be used as collateral to issue promise, $\boldsymbol{x}^b = (k)$. If the collateral requirement of the loan contracts issued at date 0 is $C^b = \phi_0 = 1/\bar{p_L}$, $F_L^i(C^b) = \phi_0 \bar{p_L} = 1$. Increasing collateral requirements will not increase r_L^b, so the collateral regulation manipulates the collateral requirement in the interval $(0, 1/\bar{p_L}]$. In the economy where the collateral of loan contracts includes both capital and CPI contracts, $\boldsymbol{x}^b = (k, \theta_j)$. The intervention is increasing h_0. When $h_0 = 1$, $\boldsymbol{F}_L^i(\boldsymbol{C}^b) = \phi_0 \bar{p_L} + h_0 r_L^j = 1$. Hence, the interval of manipulating h_0 is $[0, 1]$.

[3]Collateral hedging is costly because the sellers of CPI need to evaluate the riskiness of capital investment by small firms which do not expose much financial information. Since this paper focuses on the pecuniary externalities induced by changes in asset prices, the financial innovation cost is trivial. Thus, I assume $\varepsilon = 0$ in the following analysis.

2.3 Budget set

Given capital price (p) and financial contract prices ($(m^q)_{q\in Q}$), agents i decides commodities c_s , for each $s \in S$, investment k and contracts trades θ_{sq}. It is assumed that the expectation of price level is equal to the current price level. This means that the default of loan contracts only depends on the worth difference between promise and collateral, and is independent of the price level. At time 0, maximize utility (1) subject to the budget set defined by

$$
\begin{aligned}
B^i(p, m^q) = \{ & (c, k, \theta_q) : \\
& (c_0 + \alpha k_0 - e_0) + T(\theta^i_{0j}) + \sum_{q\in Q} \theta_{0q} m^q_0 \leq 0 \\
& c_s - e_s + p_s(k_s - k_0) + \sum_{q\in Q} \theta_{sq} m^q_s \leq \sum_{q\in Q} \theta_{0q} r^q_s, \forall s \in S_1 \\
& c_s - e_s \leq r^k_s k_{s_-} + \sum_{q\in Q} \theta_{s_-q} r^q_s, \forall s \in S_2 \\
& \sum_{b\in Q^b} max(0, -\theta_{sb}) \boldsymbol{C}^b_s \leq \boldsymbol{x}^b_s, \forall s \in \{0, L\}.
\end{aligned}
$$

The first inequality requires that money spent on consumption goods beyond the revenue from endowments and production in state 0 be financed out of the sale of contracts. The financial innovation cost function is $T(\theta^i_{0j}) = 1_{\{\theta^i_{0j}<0\}} \varepsilon \theta^i_{0j}$, where$\mathbf{1}_{\{\theta^i_j<0\}}$ is the indicator function defined on trading CPI contracts takes on the value 1 when $\theta^i_j < 0$ and 0 otherwise. The second inequality requires money spent on the consumption goods and capital goods beyond the revenue from endowments in any state $s \in S_1$ be financed out of the sale of contracts and net revenue from return from contracts bought or sold in state 0. The third inequality requires money spent on the consumption goods beyond the revenue from endowments and production in any state $s \in S_2$ be financed out of net revenue from return from contracts bought or sold in date 1. The last constraint requires that agents actually hold as least as much of collateral as financial contracts require them to hold.

2.4 Collateral equilibrium

A *collateral equilibrium* is a collection of prices, consumption, capital investment and contract trades $((p), (\bar{m^q})), (\bar{c}^i, \bar{k}, \bar{\theta_q}^i))$, such that

$$\sum_{i=1}^{3} (\bar{c_0}^i + T(\bar{\theta_j}^i) - e^i_0) + \alpha \bar{k_0} = 0 \tag{2}$$

$$\sum_{i=1}^{3} w^i \bar{k_s}^i = \bar{k_0}, \forall s \in S_1 \tag{3}$$

$$\sum_{i=1}^{3} (\bar{c_s}^i - e^i_s) - r^k_s \bar{k_{s_-}}^i = 0, \forall s \in S_2 \tag{4}$$

$$\sum_{i=1}^{3} w^i \bar{\theta_q}^i = 0, \forall q \in Q \tag{5}$$

$$(\bar{c}^i, \bar{k}, \bar{\theta_q}^i) \in B^i(\bar{m^q}), \forall i \tag{6}$$

$$(\bar{c}, \bar{k}, \bar{\theta_q}) \in B^i(\bar{p}, \bar{m^q}) \Rightarrow U^i(\bar{c}) \leq U^i(\bar{c^i}), \forall i \tag{7}$$

The first four mean that in the equilibrium markets for consumption goods, capital goods and contracts in each state clear. The equations (6) and (7) describe that agents optimize their utility subject to their budget sets. As shown in Geanakoplos and Zame (2014), the collateral equilibrium always exists under these assumptions.

I will show how agents design their portfolio in the equilibrium of the L-economy and CPI-economy in the Section 3.1.

2.5 Leverage

The key concept of this paper is leverage. The definition of leverage is the inverse of the haircut. The haircut is the complement of the LTV ratio and the definition of the LTV is a ratio of the total borrowing to the down-payment of capital investment (Geanakoplos & Zame, 2014). At date 0, with CPI contracts, optimists invest α units of consumption good to achieve 1 unit of capital investment and buy θ^1_{0j} units of CPI contracts. These serve as collateral for issuing $\mid \theta^1_{0b} \mid$ units of loan contracts. Hence, $Haircut_0$ is

$$Haircut_0 = 1 - LTV_0 = \frac{\alpha k_0 + m^j_0 \theta^1_{0j} - m^b_0 \mid \theta^1_{0b} \mid}{\alpha k_0 + m^j_0 \theta^1_{0j}}.$$

Also, the leverage at date 0 is

$$Leverage_0 = \frac{\alpha k_0 + m^j_0 \theta^1_{0j}}{\alpha k_0 + m^j_0 \theta^1_{0j} - m^b_0 \mid \theta^1_{0b} \mid}.$$

In the state L, optimists spend $p_L k^1_L$ to buy capital goods from moderates and pessimists. The capital goods serve as collateral for borrowing $m^b_L \mid \theta^1_{Lb} \mid$. Hence, LTV_L is

$$Haircut_L = 1 - LTV_L = \frac{p_L k^1_L - m^b_L \mid \theta^1_{Lb} \mid}{p_L k^1_L}.$$

The leverage in the state L is

$$Leverage_L = \frac{p_L k^1_L}{p_L k^1_L - m^b_L \mid \theta^1_{Lb} \mid}.$$

The leverage is endogenous. The $Leverage_0$ depends on capital investment and loan price and it affects the capital prices at date 1.

2.6 Discussion

This section shows the interpretation of the model and comments the role of some assumptions.

Optimists could be interpreted as firms. At date 0, they invest in capital and borrow as much as he could. They believe that they will get a great return and he is planning to pay back as he promised. However, lenders are not sure they could get paid. Moderates and pessimists as lenders in this setting, instead of checking financial information of a firm, much simpler require collateral. At date 0, because of technology barriers, they do not invest in capital and lend to agents 1. Agents 2 and 3 can be interpreted in different ways. They could be interpreted as households taking long position on corporate bonds. They could also be interpreted as financial intermediaries who can evaluate the production projects and offer funds to firms.

There are some assumptions in order to streamline the analysis. I assume that only agents 1 can invest in capital goods at date 0. Thus, capital goods are only traded at date 1. This describes a firm which produces technology or facilities at the initial date and sells them at date 1. Each agent can use the technology to produce consumption goods. Their beliefs about the state in date 2 influences capital prices which in turn determines the payoffs of loans issued at date 0.

Agents 1 could also be interpreted as commercial Banks. At date 0, they invests in risky assets to make profit. Moderates and pessimists as lenders are depositors. Even though they do not require banks hold collateral for accepting deposits directly, Basel III regulates banks to hold capital to decrease the likelihood that banks go insolvent.

3 The leverage cycle

This section firstly shows how agents behave in the L-economy and CPI-economy and then presents numerical examples of equilibrium. The results show how leverage influences capital prices positively. They raise and fall together, which is described as the leverage cycle.

3.1 Agents' behavior

3.1.1 L-economy

Table 1: Agents' behavior in the L-economy

Type	$s=0$	$s=L$	$s=H$
Optimists	Capital investment and Sell loans subject to $-\phi_0\theta^1_{0b} \leq k_0$	Sell capital and Sell loans subject to $-\phi_L\theta^1_{Lb} \leq k^1_L$	Sell capital
Moderates	Buy loans	Buy capital and Buy/Sell loans	Buy capital
Pessimists	Buy loans	Buy capital and Buy loans	Buy capital

In the L-economy, there is no CPI contracts. At date 0, agents 1 borrow money by issuing loan contracts and invest in capital which serves as collateral. Their portfolio is $\{k_0, \theta^1_{0b}\}$ where $k_0 > 0, \theta^1_{0b} < 0$. In the equilibrium, $k_0 = -\phi_0\theta^1_{0b}$, so $Leverage_0 = \phi_0\alpha/(\phi_0\alpha - m^b_0)$. Increasing ϕ_0 reduces $Leverage_0$. Agents 2 and 3 cannot get access to capital goods and only buy the loan contracts issued by agents 1. Thus, loan contract buyers whose portfolio is $\{\theta^i_{0b}\}, i = 2, 3$, where $\theta^i_{0b} > 0, i = 2, 3$.

In the state L, agents 1 repay to lenders by delivering the collateral and borrow for holding capital. Their portfolio choices are $\{k^1_L, \theta^1_{Lb}\}$, where $k^1_L > 0, \theta^1_{Lb} < 0$. In the equilibrium, $k^1_L = -\phi_L\theta^1_{Lb}$, so $Leverage_L = \phi_L p_L/(\phi_L p_L - m^b_L)$. Agents 2 and 3 will receive their loan dividends. They also buy capital goods. Agents 2 may choose to sell loans and agents 3 buy loans.[4] Their portfolio is $\{k^i_L, \theta^i_{Lb}\}, i = 2, 3$, where $k^i_L > 0, i = 2, 3$.

In the state H, agents 1 sell part of capital goods to repay the loan contract issued at date 0. Since there is no uncertainty in capital dividends, agents 1 will not borrow.[5] Their portfolio is $\{k^1_H\}$,where $k^1_H > 0$. Agents 2 and 3 buy capital goods, whose portfolio is $\{k^i_H\}, i = 2, 3$, where $k^i_H > 0, i = 2, 3$.

Table 1 shows agents' behavior in the L-economy.

[4]Even if agents 2 sell loans in the equilibrium, they are not constrained, $-\phi_L\theta^2_{0b} < k^2_L$.
[5]There is no borrowing because loans and capital goods are perfect substitutes.

3.1.2 CPI-economy

A Collateral Protection Insurance (CPI) contract which provides insurance on collateral value is issued by lenders. It pays 0 in the boom state and pays $(1 - \phi_0 \bar{p_L})$ in the bust state. CPI contracts serve as part of collateral for backing loan contracts. Thus, the delivery in the bust state is $(\phi_0 \bar{p_L} + h_0 r_L^j)$.

Table 2: Agents' behavior in the CPI-economy

Type	$s = 0$	$s = L$	$s = H$
Optimists	Capital investment, Buy CPI and Sell loans subject to $-\phi_0\theta^1_{0b} \leq k_0$ and $-h_0\theta^1_{0j} \leq \theta^1_{0j}$	Sell capital and Sell loans subject to $-\phi_L\theta^1_{Lb} \leq k^1_L$	Sell capital
Moderates	Buy loans and Sell CPI	Buy capital and Buy/Sell loans	Buy capital
Pessimists	Buy loans	Buy capital and Buy loans	Buy capital

Table 2 shows agents' behavior in the CPI-economy. Agents 1 are required to take long position of capital and CPI contracts for borrowing. At date 0, their portfolio is $\{k_0, \theta^1_{0j}, \theta^1_{0b}\}$ where $k_0 > 0, \theta^1_{0j} > 0, \theta^1_{0b} < 0$. Hence, in the equilibrium, $k_0 = -\phi_0\theta^1_{0b}$, $\theta^1_{0j} = -h_0\theta^1_{0j}$, so $Leverage_0 = (\phi_0\alpha + h_0 m^j_0)/(\phi_0\alpha + h_0 m^j_0 - m^b_0)$. Increasing h_0 reduces $Leverage_0$. Agents 2 and 3 lend to agents 1 by purchasing loan contracts. Since agents 3 require more compensation for bearing higher default risk based on their expectation than agents 2 do, agents 1 will only buy CPI contracts from agents 2 who always sell CPI contracts at a lower price than agents 3. Thus, agents 2 choose to take more risk by selling CPI contracts at date 0 and innovate. Their portfolio is $\{\theta^2_{0j}, \theta^2_{0b}\}$, where $\theta^2_{0j} < 0, \theta^2_{0b} > 0$. Moreover, CPI purchasing cannot be separate from other collateral investment. Agents 3 cannot buy CPI contracts because they do not invest in capital. Thus, agents 3 are initially constrained from trading in the CPI contracts. Their portfolio is $\{\theta^3_{0b}\}$, where $\theta^3_{0b} > 0$. In addition, CPI contracts are introduced to reduce the loss of loan buyers. If there are only two types of agents, lenders are the sellers of CPI and the buyers of loans. The increase of r^b_L they receive is paid back to borrowers in the form of CPI payoff. Thus, the setting with three types of agents is necessary, because agents 3 who do not sell CPI receive more in the bust state. Furthermore, at date 1, introducing CPI contracts is redundant, because the market is complete with two assets, namely capital goods and loan contracts. Thus, agents behave as they do in the L-economy.

In date 1, there is no collateral requirement on CPI contracts. Thus, the portfolio choices of each type of agents in the CPI economy is the same as those in the L-economy.

3.2 Numerical examples

In this section, I present three simulations based on the fundamental values in the Table 3. Since this paper focuses on how pecuniary externalities influence the effectiveness of macro-prudential policies, I assume the financial innovation cost is zero for simplicity. Since good news in the middle period reduces uncertainty about capital return, I assume that the uncertainty is 0 after state H, $r^k_{HL} = r^k_{HH}$. Since I focus on the downturn of the leverage cycle, the series of fundamental values satisfy that the capital price in the state L is less than the promise of loan contracts. Also, I assume $e^i_L < e^i_H$ to increase the difference between boom and bust in the middle period, so that the effects of macro-prudential policies on allocations are significant. Moreover, since increasing ϕ_L does not induce real changes, I assume $\phi_L r^k_{LL} = 1$ for simplicity. I also suppose a capital adjustment cost, $\alpha > 1$, to capture the investment in the industry of Industrial Machinery Manufacturing. Lastly, as I do not focus on discount rates and the weight of different groups in the economy, I suppose $\beta = 1$ and every type of agents has identical weight, $w^i = 1/3$.

Table 3: Fundamental values

	Value		Value		Value		Value
e_0^i	2	r_{HL}^k	3.8	α	1.5	w^i	0.33
e_H^i	2.5	r_{HH}^k	3.8	β	1	π_L^2	0.5
e_L^i	1.02	r_{LH}^k	3.8	ϕ_L	1.43	ε	0
$e_{s_2}^i$	1.02	r_{LL}^k	0.7	ρ	1		

The first simulation shows how the asset prices and leverage engage in a positive feedback and compare the leverage cycles with different belief disagreement. The second simulation shows how leverage and capital price in the state L are affected by belief disagreement. I do the third simulation to compare the effects of the macro-prudential policies on the leverage cycle in the environment with varying belief disagreement (σ). The policy target is $r_L^b = 1$. Specifically, the policy tools are (i) increasing the collateral requirement on capital by $\Delta\phi_0$; (ii)introducing CPI contracts and increasing the collateral requirement on CPI contracts by Δh_0.

In this section, I specialize the model to the case of an isoelastic utility function with intertemporal elasticity of substitution ρ:

$$u_s^i = \begin{cases} \frac{(c_s^i)^{1-\rho}}{1-\rho} & if\ \rho \neq 1 \\ log(c_s^i) & if\ \rho = 1 \end{cases}$$

Additionally, to simplify the belief distribution, I suppose that skewness is zero and that π_s^2 is identical in each distribution. For example, beliefs are given by $\pi_L^1 = \pi_L^2 - \sigma$, π_L^2 and $\pi_L^3 = \pi_L^2 + \sigma$. The each type of agents' belief is ranked as $\pi_L^1 < \pi_L^2 < \pi_L^3$. Notice that this study does not change fundamentals such as utility functions, endowments, capital returns and asset payoffs in the three simulations.

3.2.1 An example

Table 4: Equilibrium Leverage Cycle when $\sigma = 0.15$

States	$s=0$	$s=H$	$s=L$
m_s^b	0.9335	-	0.6226
$Leverage_s$	2.6478	-	1.8000
p_s	1.5	3.4275	0.9807
k_s	1.3829	0.7202	0.5381

Table 5: Equilibrium Leverage Cycle when $\sigma = 0.25$

States	$s=0$	$s=H$	$s=L$
m_s^b	0.9123	-	0.6512
$Leverage_s$	2.5524	-	1.9060
p_s	1.5	3.3161	0.9589
k_s	1.4565	0.7514	0.6809

Simulation 1 provides an example of the leverage cycle with an additional parameter values, the collateral requirement of loan contracts issued at date 0, $\phi_0 = 1$. The reason why I choose $\phi_0 = 1$ is that it is higher than the collateral requirement which does not constrain optimists. [6]Also, with $\phi_0 = 1$, in the equilibrium, $r_L^b < 1$ calls for increasing collateral requirements.

At date 0, it takes 1.5 units of consumption goods to achieve 1 unit of capital goods. Table 4 and 5 show that at date 1, capital prices are rising and falling with the leverage. There are three reasons for the crash of capital prices. Firstly, from state 0 to the state L, the uncertainty is increased. For example, when $\sigma = 0.15$, pessimists believe the possibility of state LL is 0.4225 at date 0, but at the state L, the possibility is 0.65. The pessimists value the capital goods much less. The disagreement

[6]The not binding collateral requirement is 0.5351, when $\sigma = 0.15$. Also, in the Simulation 3, this fixed $\phi = 1$ satisfies this condition in the economy with varying belief disagreement $\sigma \in [0.05, 0.25]$.

between optimists and pessimists over the capital value is higher at L than at 0. Optimists believe that the low capital price is an opportunity, because the expected payoff of capital goods reduces little. They will borrow with loan contracts which generate higher return. It is shown in the first row in table 4 that $m_0^b > m_L^b$. Also, k_0 is more in the case where belief disagreement is 0.25. It implies that optimists are willing to take more risk if π_H^1 is larger.

The second reason is deleveraging. Since the value of capital goods is composed into two parts, namely the discounted future payoff and its pledge-ability. The lower leverage at state L means that the borrowing which capital goods support is less. Thus, the capital price goes down. The third reason is the bankruptcy of optimists. They are capital buyers. Their wealth loss weakens their purchasing power, resulting in lower capital price. Table 4 also shows that when k_0 is higher, the loss of optimists is higher. The p_L is lower.

3.2.2 Asset prices and leverage for varying belief heterogeneity

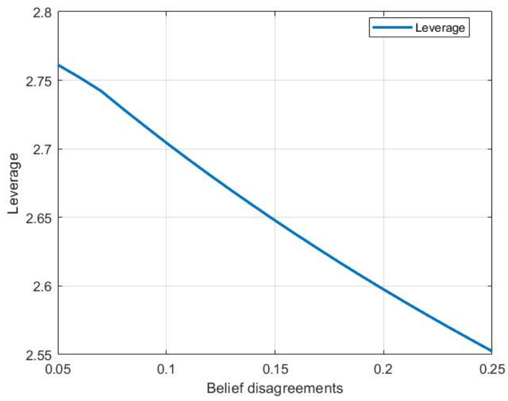

Figure 4: Leverage for varying σ when $\phi_0 = 1$

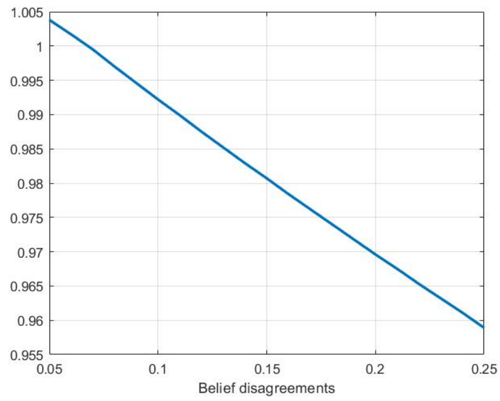

Figure 5: p_L for varying σ when $\phi_0 = 1$

Since the capital price determines default of loan contracts, simulation 2 examines varying belief disagreement with the collateral requirement fixed. Figure 6 demonstrates that if the belief heterogeneity is high, leverage at date 0 is low. It means the borrowing capacity of agents 1 is low, if lenders consider the collateral value reducing. Figure 7 shows that if belief disagreement is larger, p_L is lower. Lenders lose more in the state L. Also, when belief disagreement is extremely low, capital price in the L state is higher than 1. This means that though capital returns after state L are uncertain, loan contract sellers can deliver promised 1 by selling capital. It implies that if agents share similar belief, capital price falls little in the state L. The macro-prudential policies are not necessary to protect lenders. The results in the two figures are consistent with the argument of Geanakoplos and Zame (2014).

3.2.3 Interpreting stylized facts in the model

The above numerical exercises illustrate equilibrium outcomes in the absence of macro-prudential interventions. This section focuses on the effects of two macro-prudential policies: (i) increasing the collateral requirement on capital by $\Delta\phi_0$; (ii)introducing CPI contracts and increasing the collateral requirement on CPI contracts by Δh_0. The following simulation results provide a way to jointly interpret the qualitative patterns of the stylized facts described in the Section 1.

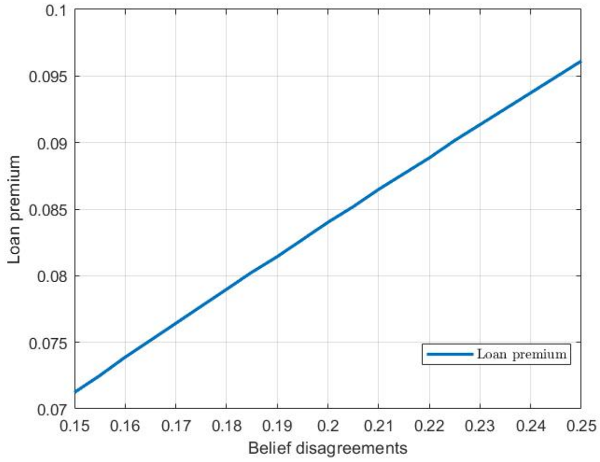

Figure 6: Loan premium for varying σ

Rationalization of Fact 1 Fact 1 describes the changes in ABCP spread during the global financial crisis. In this model, the ABCP spread is indexed by the loan premium, which is defined as the promised return over the loan price $1/m^b$. Figure 6 shows that when the belief disagreement is larger and the loan premium is higher. This is qualitatively consistent with Fact 1. Before the global financial crisis, agents receive bad news and update their beliefs about final payoffs. When loan contract buyers hold much more pessimistic beliefs than loan contract sellers do, the promises of borrowers are valued much less by lenders. Borrowers should offer higher returns to sell ABCPs. Thus, the ABCP spread reflect the belief heterogeneity in the economy.

Table 6: Effects of two macro-prudential policies when $\sigma = 0.15$

	Benchmark	$\Delta\phi_0$	Δh_0
r_L^b	0.9696	1	1
$(p_L - \alpha)/\alpha$	-0.3462	-0.3457	-0.3464
$(c_L^1 - c_0^1)/c_0^1$	-0.4026	-0.4016	-0.4026
$(c_L^2 - c_0^2)/c_0^2$	-0.1728	-0.1752	-0.1746
$(c_L^3 - c_0^3)/c_0^3$	-0.1030	-0.1037	-0.1004
V	1.6149	1.6147	1.6154

Table 7: Effects of two macro-prudential policies when $\sigma = 0.25$

	Benchmark	$\Delta\phi_0$	Δh_0
r_L^b	0.9589	1	1
$(p_L - \alpha)/\alpha$	-0.3607	-0.3599	-0.3615
$(c_L^1 - c_0^1)/c_0^1$	-0.4079	-0.4057	-0.4080
$(c_L^2 - c_0^2)/c_0^2$	-0.1603	-0.1665	-0.1651
$(c_L^3 - c_0^3)/c_0^3$	-0.0517	-0.0515	-0.0438
V	1.6650	1.6656	1.6666

Rationalization of Facts 2 and 3 Fact 2 describes that CCAR stress tests serving as leverage restriction reduced the volatilities of asset prices and increased the real personal consumption expenditures. In contrast, Fact 3 describes that IGLL serving as leverage restriction reduced the volatilities of asset prices but reduce the real personal consumption expenditures. As previously noted, the belief heterogeneity was different when these two macro-prudential policies became effective. When CCAR stress tests took effect, belief heterogeneity was large because in 2011, the lack of fundamental changes in banking and financial markets worried market participants. However, when IGLL took effect in 2014, belief heterogeneity was small. As policy announcements about quantitative easing in 2012 stimulated the economy, most market participants recovered.

In this context, leverage restriction is characterized by increasing ϕ_0. The volatility of asset prices is indexed by differences of asset prices between state 0 and state L, $(p_L - \alpha)/\alpha$. The real personal consumption expenditure level is positively related to social welfare, V. The model simulation results in Tables 6 and 7 present the effect of macro-prudential policies $\Delta\phi_0 > 0$ targeting $r_L^b = 1$ when belief disagreement is $\sigma = 0.15$ or $\sigma = 0.25$ respectively. The second row in tables 6 and 7 show that $\Delta\phi_0$ reduces the difference of asset prices. The last row in Table 6 shows that when belief disagreement is 0.15, increasing ϕ_0 by 0.0189 reduces social welfare. By contrast, Table 7 presents that when belief disagreement is 0.2, macro-prudential policies $\Delta\phi_0 = 0.0415$ increase social welfare. The results imply that the level of belief disagreement influences the effect of the macro-prudential policies, which is consistent with the two stylized facts.

Rationalization of Fact 4 Fact 4 describes the higher capital charge on G-SIB increases the volatilities of asset prices and increases the real personal consumption expenditures. This macro-prudential policies require financial institutions to adjust their asset components for lower risk exposure and resiliency improvement. In this model, CPI contracts as an additional type of collateral reduces loss if a financial institution failed to repay in the crisis. The simulation results in Table 6 and 7 show that increasing h leads to a larger decline of capital prices. CPI substitutes capital goods as part of collateral, leading asset prices to decrease. However, the more decrease of capital prices does not induce a drop in social welfare. This implies that the lower social loss of G-SIB default results in welfare improvement.

3.2.4 Effects of macro-prudential policies on the volatilities of consumption

To figure out the reason for the welfare changes, I look into the difference of consumption between state 0 and state L.[7]Table 6 shows that increasing ϕ_0 induces the consumption differences of agents 1 to decrease and the consumption differences of agents 2 and 3 to increase. By contrast, a higher h_0 drives the consumption of agents 1 and 2 up and the consumption differences of agents 3 down. This implies that the consumption differences of agents 3 (pessimists) is dominant in the effect of macro-prudential policies on social welfare. The reason is that pessimists have a strong desire to transfer wealth from states with high consumption to periods with low consumption. As increasing h_0 stabilizes consumption of agents 3 in the cycle, it improves social welfare. This is also proved by the results in the first and third columns of table 7. Introducing CPI contracts provides an extra insurance instrument against the default in the recession. It increases social welfare by reducing pessimists' consumption difference between state 0 and state L. [8]Moreover, the comparison between the first and second column in the table 7 illustrates that social welfare is improved by increasing ϕ_0 and the consumption difference of agents 3 decreases. This motivates the following discussion about the mechanism of how macro-prudential policies affects the capital prices and then social welfare.

3.2.5 Effects of macro-prudential policies on the leverage cycle

To increase the loan contract return in the state L, policy makers can increase ϕ or introduce CPI contracts and increase h. The results in the Tables 8 and 9 show the effect of the macro-prudential policies in the economy where $\sigma = 0.15$. When $\phi_0 = 1.0189$, $\bar{p_L} = 1/\phi_0$. Lenders receive the promised delivery at the state L. Comparing Tables 4 and 8, I find the m_0^b is higher and the capital investment is lower with a higher ϕ_0. It reflects that tightening collateral constraints increases the funding cost and reduce investment in the production activities. However, the results in Tables 4 and 9 show that with a

[7]Dogra and Gorbachev (2016) argued that consumption volatility increased significantly between 1983 and 2007, indicating substantial welfare losses.

[8](Lopez et al., 2016) proposed that the central bank should focus on removing consumption volatility.

Table 8: The effects of $\Delta\phi_0$ on Equilibrium Leverage Cycle when $\sigma = 0.15$

States	$s = 0$	$s = H$	$s = L$
m_s^b	0.9490	-	0.6230
$Leverage_s$	2.6379	-	1.7997
p_s	1.5	3.4309	0.9815
k_s	1.3807	0.7230	0.5374

Table 9: The effects of Δh_0 on Equilibrium Leverage Cycle when $\sigma = 0.15$

States	$s = 0$	$s = H$	$s = L$
m_s^b	0.9457	-	0.6226
$Leverage_s$	2.6701	-	1.8004
p_s	1.5	3.4264	0.9803
k_s	1.3836	0.7205	0.5386

positive collateral requirement on CPI contracts, the m_0^b is higher and the capital investment is higher. Though the loan price is higher in both cases, the capital investment increases in the CPI-economy with cheaper collateral.

Additionally, in the L-economy, with an increase on ϕ_0, the capital investment decreases and the capital prices in the states H and L increase. The leverage in both state 0 and state L is lower than the leverage in the benchmark case. Also, the difference in leverage between 0 and L is 31.77%, which is lower the difference (32.02%) in the benchmark case. By contrast, increasing h leads capital investment to increase and drives the capital prices down. The reason for negative effects of increasing h on capital prices is the higher capital investment at date 0. When there are more capital goods in the economy, agents will value them less. The leverage increases in the state 0 and L. The difference in leverage is 32.57%, which is higher than that in the benchmark case. This higher leverage and the higher difference in leverage imply that the introduction of CPI contracts may hurt stability, which may result in a decrease in social welfare.

Since increasing collateral requirements on capital reduces the leverage, it captures leverage restrictions like the CCAR stress tests and the IGLL and FAQs. The results in the previous section also explains the first questions raised by the stylized fact 1. Furthermore, introduction of CPI also provides an example of a macro-prudential policy that increase the volatility of leverage and asset prices but improves social welfare.

Table 10: The effects of $\Delta\phi_0$ on Equilibrium Leverage Cycle when $\sigma = 0.25$

States	$s = 0$	$s = H$	$s = L$
m_s^b	0.9458	-	0.6519
$Leverage_s$	2.5341	-	1.9059
p_s	1.5	3.3215	0.9601
k_s	1.4528	0.7585	0.6797

Table 11: The effects of Δh_0 on Equilibrium Leverage Cycle when $\sigma = 0.25$

States	$s = 0$	$s = H$	$s = L$
m_s^b	0.9385	-	0.6512
$Leverage_s$	2.5994	-	1.9081
p_s	1.5	3.3126	0.9577
k_s	1.4588	0.7524	0.6829

Tables 10 and 11 shows how the macro-prudential policies influences the leverage cycle in the economy where the belief disagreement is 0.25. With a higher ϕ_0, the leverage difference between state 0 and state L is 24.79%, which is reduced from the difference (25.32%) in the benchmark case. However, increasing h_0 raise the difference to 26.60%. These results also reflect the discussion above.

3.3 Robustness analysis

In the previous section, three simulations fix the initial collateral requirement on capital goods at 1. In order to verify if this specification is robust, I do the three simulations with $\phi_0 = 0.8$. The results are shown in the Tables 12-19. Tables 12 and 13 show that capital prices are increasing and decreasing with leverage. Also, a higher investment at date 0 induces a lower capital prices at date 1. Tables 14 and 15 illustrate that the macro-prudential policy $\Delta\phi_0 > 0$ leads the difference of capital prices between state 0 and state L to decline, while $\Delta h_0 > 0$ leads the difference to increase. Also, the results related to consumption differences and social welfare show that the decrease of the consumption volatility of pessimists induces an increase of social welfare. Moreover, Tables 16 to 18 verify that the results in tables 8-11 are robust. I also test the simulations with ϕ_0 fixed at 0.85 and 0.9 and ρ. The results has similar characteristics. I also test the simulations with different risk preferences, $\rho = 0.8$ and $\rho = 2$. In short, the robustness analysis generates the same intuition as the above discussion.

4 Pecuniary externalities and macro-prudential perturbations

The previous section has shown how the macro-prudential tools affect the leverage cycle in the L-economy and the CPI-economy. In this section, I present how the changes of capital prices induce the pecuniary externalities which influence the effect of macro-prudential perturbations on social welfare.

4.1 Macro-prudential perturbations in the L-economy

The tightness of collateral constraint in the L-economy is captured by the Lagrangian multiplier of the collateral constraint on capital which is represented by $\bar{\mu}^1$. Moreover, the different beliefs result in an inequality of the agents' present values of future wealth. The present values $\gamma_s^i = \lambda_s^i/\lambda_0^i, s \in S_T$, where $\lambda_s^i, s \in S$ is the marginal utilities of consumption. Thus, with the assumption $\pi_L^1 < \pi_L^2 < \pi_L^3$, in the incomplete market, the present values in the original equilibrium is ranked as $\bar{\gamma_L}^1 < \bar{\gamma_L}^2 < \bar{\gamma_L}^3$ and $\bar{\gamma_H}^1 > \bar{\gamma_H}^2 > \bar{\gamma_H}^3$

In the L-economy, the perturbation $d\phi_0$ at the initial state, where $d\phi_0 > 0$ induce marginal changes at date 0, $(dc_0^i, dk_0, d\theta_{0b}^i)$,with $\sum\limits_{i=1}^{3} w^i d\theta_{0b}^i = 0$, and then adjustments of the subsequent equilibrium plans and prices $(dc_s^i, dk_L^i, dk_H^i, , dm_0^b, dm_L^b, dp_L, dp_H)$ around the equilibrium $(\bar{c_s}^i, \bar{k_0}, \bar{k_L}^i, \bar{k_H}^i, \bar{p_H}, \bar{p_L}, \bar{\theta_{0b}}^i, \bar{\theta_{Lb}}^i, \bar{m_0^b}, \bar{m_L^b})$. Assume $\boldsymbol{C}^b = (\bar{\phi_0})$. The complete derivation is in Appendix C.I.

If in the equilibrium, $\bar{p_L} < 1/\bar{\phi_0}, \bar{p_H} > 1/\bar{\phi_0}$, agents 1 default in the state L. The change in social welfare is

$$\left(\sum_{i=1}^{3} w^i \frac{du^i}{\bar{\lambda_0}^i} \mid_{\phi_0 = \bar{\phi_0}}\right)/d\phi_0 = \Gamma_L^{21} + \Gamma_L^{31} + \Phi_0^1 + P_L^{\phi,b} + P_H^{\phi,k} + P_L^{\phi,k}, \tag{8}$$

where I refer to Γ_L^{i1} as the heterogeneous effects between agents i and agents 1, $i = 2, 3$

$$\Gamma_L^{i1} = w^i \bar{\theta_{0b}}^i (\bar{\gamma_L}^i - \bar{\gamma_L}^1)\bar{p_L}, \tag{9}$$

and I refer to Φ_0^1 as the collateral effects induced by the perturbation on the collateral requirement at date 0

$$\Phi_0^1 = w^1 \bar{\theta_{0b}}^1 \frac{\bar{\mu_0}^1}{\bar{\lambda_0}^1}, \tag{10}$$

and I refer to $P_s^{\phi,b}$ and $P_s^{\phi,k}$ as the pecuniary externalities induced by the changes of loan contract price and capital prices at state s respectively,

Table 12: Equilibrium Leverage Cycle when $\sigma = 0.15$

States	$s = 0$	$s = H$	$s = L$
m_s^b	0.7691	-	0.6183
$Leverage_s$	2.7851	-	1.8026
p_s	1.5	3.3911	0.9721
k_s	1.4064	0.6784	0.5457
c_s^1	1.2425	3.2108	0.7257

Table 13: Equilibrium Leverage Cycle when $\sigma = 0.25$

States	$s = 0$	$s = H$	$s = L$
m_s^b	0.7506	-	0.6477
$Leverage_s$	2.6705	-	1.9064
p_s	1.5	3.2914	0.9535
k_s	1.4734	0.7025	0.6859
c_s^1	1.1724	3.1956	0.6769

Table 14: Effects of two macro-prudential policies when $\sigma = 0.15$

	Benchmark	$\Delta\phi_0$	Δh_0
r_L^b	0.7777	1	1
$(p_L - \alpha)/\alpha$	-0.3519	-0.3457	-0.3546
$(c_L^1 - c_0^1)/c_0^1$	-0.4159	-0.4016	-0.4165
$(c_L^2 - c_0^2)/c_0^2$	-0.1417	-0.1752	-0.1676
$(c_L^3 - c_0^3)/c_0^3$	-0.0944	-0.1037	-0.0591
V	1.6163	1.6147	1.6217

Table 15: Effects of two macro-prudential policies when $\sigma = 0.25$

	Benchmark	$\Delta\phi_0$	Δh_0
r_L^b	0.7628	1	1
$(p_L - \alpha)/\alpha$	-0.3643	-0.3599	-0.3691
$(c_L^1 - c_0^1)/c_0^1$	-0.4226	-0.4057	-0.4236
$(c_L^2 - c_0^2)/c_0^2$	-0.1245	-0.1665	-0.1587
$(c_L^3 - c_0^3)/c_0^3$	-0.0527	-0.0515	-0.002
V	1.6594	1.6656	1.6685

Table 16: The effects of $\Delta\phi_0$ on Equilibrium Leverage Cycle when $\sigma = 0.15$

States	$s = 0$	$s = H$	$s = L$
m_s^b	0.9490	-	0.6230
$Leverage_s$	2.6379	-	1.7997
p_s	1.5	3.4309	0.9815
k_s	1.3807	0.7230	0.5374
c_s^1	1.2149	3.4015	0.9815

Table 17: The effects of Δh_0 on Equilibrium Leverage Cycle when $\sigma = 0.15$

States	$s = 0$	$s = H$	$s = L$
m_s^b	0.9068	-	0.6182
$Leverage_s$	3.1152	-	1.8084
p_s	1.5	3.3796	0.9681
k_s	1.4139	0.6811	0.5511
c_s^1	1.2433	3.2091	0.7249

Table 18: The effects of $\Delta\phi_0$ on Equilibrium Leverage Cycle when $\sigma = 0.25$

States	$s = 0$	$s = H$	$s = L$
m_s^b	0.9458	-	0.6519
$Leverage_s$	2.5341	-	1.9059
p_s	1.5	3.3215	0.9601
k_s	1.4528	0.7585	0.6797
c_s^1	1.1401	3.4110	0.6776

Table 19: The effects of Δh_0 on Equilibrium Leverage Cycle when $\sigma = 0.25$

States	$s = 0$	$s = H$	$s = L$
m_s^b	0.8990	-	0.6478
$Leverage_s$	3.0183	-	1.9199
p_s	1.5	3.2718	0.9464
k_s	1.4870	0.7073	0.6988
c_s^1	1.1721	3.1924	0.6755

$$P_L^{\phi,b} = \sum_{i=1}^{3} w^i \bar{\gamma_L}^i (-\bar{\theta_{Lb}}^i \frac{dm_L^b}{d\phi_0}), \tag{11}$$

$$P_H^{\phi,k} = (w^1 \bar{\gamma_H}^1 \bar{k_0} + \sum_{i=1}^{3} w^i \bar{\gamma_H}^i (-\bar{k_H}^i)) \frac{dp_H}{d\phi_0}, \tag{12}$$

$$P_L^{\phi,k} = (w^2 \bar{\gamma_L}^2 \bar{\theta_{0b}}^2 \bar{\phi_0} + w^3 \bar{\gamma_L}^3 \bar{\theta_{0b}}^3 \bar{\phi_0} - \sum_{i=1}^{3} w^i \bar{\gamma_L}^i \bar{k_L}^i) \frac{dp_L}{d\phi_0}. \tag{13}$$

The effect of a positive $d\phi_0$ on social welfare is determined by the sign of equation (8). Equations (9) and (10) reflect the effect of the perturbation $d\phi_0 > 0$ on social welfare through loan contract trading at date 0. In the equation (9), $\bar{p_L} d\phi_0$ describes the increase of r_L^b induced by a higher collateral requirement. $\bar{\gamma_L}^i \bar{p_L} d\phi_0$ represents the present value of the higher r_L^b for agents i. As agents hold different beliefs and $\bar{\gamma_L}^1 < \bar{\gamma_L}^2 < \bar{\gamma_L}^3$, the sign of equation (9) is positive. Equation (9) characterizes the heterogeneity effect which increases social welfare.

In the equation (10), $\frac{\bar{\mu}^1}{\bar{\lambda_0}^1}$ means the substitution rate between the value of holding capital as collateral and the marginal utility of consumption at the initial date. A higher collateral requirement induces the needs for more collateral to satisfy borrowing needs, $w^1 \bar{\theta_{0b}}^1$. As $\bar{\theta_{0b}}^1 < 0$, the sign of equation (10) is negative. It characterizes collateral effect which decreases social welfare.

The equation (11) captures the channel of loan contract trading at state L. $\bar{\theta_{Lb}}^i dm_L^b$ stands for the effect of perturbation $d\phi_0 > 0$ on money lent or borrowed by trading loans. As $\bar{\gamma_L}^1 < \bar{\gamma_L}^2 < \bar{\gamma_L}^3$, the positive sign of the equation (11) requires $dm_L^b < 0$. This means that reducing the cost of issuing loan contracts is a positive force of the perturbation, $d\phi_0 > 0$ on social welfare.

The equations (12) and (13) show how pecuniary externalities induced by capital price changes $dp_s, s \in S_1$ affect social welfare. $\bar{k_s}^i dp_s, s \in S_1$ describes how the capital trading at date 1 influences social welfare. $\bar{k_0} dp_H$ is the changes of the return of capital investment. As $\bar{\gamma_H}^1 > \bar{\gamma_H}^2 > \bar{\gamma_H}^3$, the positive sign of the equation (12) requires $dp_H > 0$. Moreover, in the equation (13), $\bar{\theta_{0b}}^i \bar{\phi_0} dp_L, i = \{2, 3\}$ characterizes the changes of r_L^b by the changes of capital price in state L. If the $w^2 \leq w^3$ and $dp_L > 0$, the sign of equation (13) is positive. These two conditions reflect that there are positive pecuniary externalities if increasing ϕ_0 leads capital price in the states H and L to rise.[9]

Proposition 1. *When there is default in the state L, the sign of the effect of the perturbation $d\phi_0 > 0$ on social welfare is determined by the sign and magnitude of the following effects and externalities:*

i) Heterogeneity effect: the sign is positive and the magnitude is determined by the difference in the present values, $\bar{\gamma_L}^i$ and the net trading positions of loan contracts, $\bar{\theta_{0b}}^i, i = 2, 3$ and the capital price, $\bar{p_L}$;

ii) Collateral effect: the sign is negative and the magnitude of collateral effects is determined by the cost of holding collateral for borrowing needs, $\bar{\theta_{0b}}^1 \frac{\bar{\mu_0}^1}{\bar{\lambda_0}^1}$;

iii) Pecuniary externalities: The sign and magnitude of pecuniary externalities are determined by the following variables: 1) The difference in the present value $\bar{\gamma_s}^i, s \in S_1$; 2) The net trading positions on capital $k_s^i, s \in S_1$ and loan contracts θ_{0b}^i and θ_{Lb}^i; 3) The sensitivity of equilibrium prices to the perturbation $d\phi_0 > 0$, $\frac{dm_L^b}{d\phi_0}, \frac{dp_s}{d\phi_0}, s \in S_1$.

Proposition 1 shows the information needed to determine whether increasing the collateral requirement on capital improves social welfare. When borrowers default in the state L, heterogeneity effects arise. As the higher collateral requirement increases the loan return in the bust state, lenders' utilities

[9]The proof is in Appendix C.

go up while borrowers utilities go down. Since the present values of lenders are higher than borrowers', the increase of lenders' utilities is more than the decrease of borrowers' utilities. Also, the capital price in the state L of the initial equilibrium and lenders' net positions of loans affects the magnitude of heterogeneity effects.

Collateral effects arise because the macro-prudential perturbation $d\phi_0 > 0$ directly influences the borrowing capacity of borrowers at date 0. Thus, relaxing borrowers' collateral constraints induce welfare benefit because it improves the effective financial decisions of constrained agents. The equation (10) also reflects that only agents 1 are affected by collateral externalities, while all of agents are affected by heterogeneity effects.

Pecuniary externalities arise for two reasons: (i) agents hold heterogeneous beliefs; (ii) the perturbation $d\phi_0 > 0$ impacts capital prices and the funding cost at date 1. If tightening collateral constraints of capital increases capital prices, this perturbation increases present values of capital holders at date 1. In the state H, agents 2 and 3 buy capital goods from agents 1. The revenue of selling capital increased by the perturbation $d\phi_0 > 0$ improves the agents 1 utility. As agents 1 are optimists, they have higher present value γ_H^1 than agents 2 and 3. Thus, a higher capital price has a positive effect on social welfare. In addition, in the state L, agents 1 should deliver collateral, namely capital goods to agents 2 and 3 as the repayment of loans. The return of lenders increases if the capital price in the state L is raised by the higher collateral requirement. Although the higher capital price reduces the utilities of agents 1 in the state L, this negative force is less than the positive force induced by the increase of lenders' utilities. Moreover, if the higher collateral requirement increases the loan contract price in the state L, agents 1 promise less return for borrowing, resulting in a negative effect on social welfare. The lower funding cost of constrained agents reduces lenders' utilities, when the capital return is uncertain in the state L. Also, the results in the table 4 and 6 show that increasing collateral requirement can induce both a higher p_L and m_L^b. It means that the funding should be costly in a more uncertain situation though the price of collateral is higher.

The results in table 10 present that social welfare decreases with a higher ϕ_0. With the fundamental values in the table 3, when belief disagreement is 0.15 or 0.25, the heterogeneity effect, collateral effect and the pecuniary externalities are shown in the following table. Compare the sizes of the following effects or pecuniary externalities, tables 20 and 21 show that heterogeneity effects and collateral effects are dominant.

Table 20: The effects of perturbations, $d\phi_0$, when $\sigma = 0.15$

Γ_L^{21}	Γ_L^{31}	Φ_0^1
0.0040	0.0330	-0.0579
$P_L^{\phi,b}$	$P_H^{\phi,k}$	$P_L^{\phi,k}$
-0.0005	-0.0019	0.0010

Table 21: The effects of perturbations, $d\phi_0$, when $\sigma = 0.25$

Γ_L^{21}	Γ_L^{31}	Φ_0^1
0.0372	0.0924	-0.1232
$P_L^{\phi,b}$	$P_H^{\phi,k}$	$P_L^{\phi,k}$
-0.0014	0.0002	0.0026

The decline can be explained by a small welfare improvement induced by the heterogeneous effect. This is also reflected by a higher consumption differences between state 0 and state L of moderates and pessimists. However, if there is a strong heterogeneous effect, the social welfare may be increased. For example, Table 11 provides the evidence that when the consumption differences between state 0 and state L of moderates decrease, social welfare is improved by a higher ϕ_0.

If in the equilibrium, $\bar{p_L} > 1/\bar{\phi_0}, \bar{p_H} > 1/\bar{\phi_0}$, agents 1 do not default in the state L. The change in social welfare is

$$(\sum_{i=1}^{3} w^i \frac{du^i}{\bar{\lambda}_0^i} |_{\phi_0=\bar{\phi_0}})/d\phi_0 = \Phi_0^1 + P_L^{\phi,b} + P_H^{\phi,k} + P_{n,L}^{\phi,k} \tag{14}$$

where I refer to $P_{n,L}^{\phi,k}$ as the pecuniary externalities induced by capital price changes in the case where there is no default in state L,

$$P_{n,L}^{\phi,k} = (w^1 \bar{\gamma_L}^1 \bar{k_0} + \sum_{i=1}^{3} w^i \bar{\gamma_L}^i (-\bar{k_L}^i)) \frac{dp_L}{d\phi_0}. \tag{15}$$

The equation (14) shows that if the loan contracts sellers do not default in the state L. The perturbation $d\phi_0 > 0$ does not induce heterogeneity effect. The reason is that when $r_L^b = min\{1, \bar{\phi_0}\bar{p_L}\} = 1$, a higher ϕ_0 does not increases lenders' utility or reduce borrowers' utility in the state L. Moreover, the perturbation still induces collateral effect and pecuniary externalities, but pecuniary externalities induced by capital price changes dp_L are different. In the equation (15), $\bar{k_0} dp_L$ characterizes the changes of the return of capital investment and $\bar{k_L}^i dp_L$ describes the trading of capital goods. As $\bar{\gamma_L}^1 < \bar{\gamma_L}^2 < \bar{\gamma_L}^3$, the positive sign of the equation (15) is $dp_L < 0$. This means that if agents 2 and 3 spend less in buying capital goods from agents 1 in the state L, the perturbation $d\phi_0 > 0$ has a positive force on welfare improvement by reducing p_L.

Corollary 1. *When there is no default in the state L, the sign of the effect of the perturbation $d\phi_0 > 0$ on social welfare is determined by the sign and magnitude of the following effects and externalities:*

i) Collateral effect: the sign is negative and the magnitude of collateral effects is determined by the cost of holding collateral for borrowing needs, $\bar{\theta_{0b}}^1 \frac{\bar{\mu_0}^1}{\bar{\lambda_0}^1}$;

ii) Pecuniary externalities: The sign and magnitude of pecuniary externalities are determined by the following variables: 1) The difference in the present value $\bar{\gamma_s}^i, s \in S_1$; 2) The net trading positions on capital $k_s^i, s \in S_1$ and loan contracts θ_{0b}^i and θ_{Lb}^i; 3) The sensitivity of equilibrium prices to the perturbation $d\phi_0 > 0$, $\frac{dm_L^b}{d\phi_0}, \frac{dp_s}{d\phi_0}, s \in S_1$.

4.2 Macro-prudential perturbations in the CPI-economy

In the CPI-economy, the perturbation dh_0 at the initial state, where $dh_0 > 0$ induce marginal changes at date 0, $(dc_0^i, dk_0, d\theta_{0b}^i, d\theta_{0j}^i)$,with $\sum_{i=1}^{3} w^i d\theta_{0b}^i = 0$, $\sum_{i=1}^{3} w^i d\theta_{0j}^i = 0$ and then adjustments of the subsequent equilibrium plans and prices $(dc_s^i, dk_L^i, dk_H^i, dm_0^b, dm_0^j, dm_L^b, dp_L, dp_H)$ around the equilibrium $(\bar{c_s}^i, \bar{k_0}, \bar{k_L}^i, \bar{k_H}^i, \bar{p_H}, \bar{p_L}, \bar{\theta_{0b}}^i, \bar{\theta_{0j}}^i, \bar{\theta_{Lb}}^i, \bar{m_0^b}, \bar{m_0^j}, \bar{m_L^b})$ and $\bar{p_L} < 1/\bar{\phi_0}, \bar{p_H} > 1/\bar{\phi_0}$. $\boldsymbol{C}_0^b = (\bar{\phi_0}, \bar{h_0})$, where $\bar{\phi_0} \in (0, 1/\bar{p_L})$ and $\bar{h_0} \in (0, 1]$. The complete derivation is in Appendix C.II. The change in social welfare is

$$(\sum_{i=1}^{3} w^i \frac{du^i}{\bar{\lambda}_0^i} |_{h_0=\bar{h_0}})/dh_0 = \mathit{\Gamma}_L^{32} + P_L^{h,b} + P_H^{h.k} + P_L^{h,k} + \Upsilon_0, \tag{16}$$

where I refer to $\mathit{\Gamma}_L^{32}$ as the heterogeneous effect between agents 3 and agents 2

$$\Gamma_L^{32} = w^3 \bar{\theta_{0b}}^3 (\bar{\gamma_L}^3 - \bar{\gamma_L}^2) r_L^j, \tag{17}$$

and I refer to $P_s^{\phi,b}$ and $P_s^{\phi,k}$ as the pecuniary externalities induced by the changes of loan contract price and capital prices at state s respectively,

$$P_L^{h,b} = \sum_{i=1}^{3} w^i \bar{\gamma_L}^i (-\bar{\theta_{Lb}}^i \frac{dm_L^b}{dh_0}), \tag{18}$$

$$P_H^{h,k} = (w^1 \bar{\gamma_H}^1 \bar{k_0} + \sum_{i=1}^{3} w^i \bar{\gamma_H}^i (-\bar{k_H}^i)) \frac{dp_H}{dh_0}, \tag{19}$$

$$P_L^{h,k} = ((w^2 \bar{\gamma_L}^2 \bar{\theta_{0b}}^2 + w^3 \bar{\gamma_L}^3 \bar{\theta_{0b}}^3 + \bar{h_0} \bar{\theta_{0b}}^3 (\bar{\gamma_L}^3 - \bar{\gamma_L}^2)) \bar{\phi_0} - \sum_{i=1}^{3} w^i \bar{\gamma_L}^i \bar{k_L}^i) \frac{dp_L}{dh_0}. \tag{20}$$

and I refer to Υ_0 as the financial innovation cost effect induced by introducing CPI contracts,

$$\Upsilon_0 = w^1 \bar{\theta_{0b}}^1 \varepsilon \tag{21}$$

The positive sign of equation (16) stands for a positive effect of the perturbation $dh_0 > 0$ on welfare. The equation (17) characterizes the channel of CPI contract trading via which the perturbation $dh_0 > 0$ impacts social welfare. In the CPI-economy, the default of agents 1 lead agents 2 to lose and agents 3 is protected by CPI trading. The transfer of the present value loss from agents 3 to 2 is characterized equation (17). Since $\bar{\gamma_L}^3 > \bar{\gamma_L}^2$, the sign of the equation (17) is positive.

The equations (18) and (19) captures pecuniary externalities induced by loan contract trading at state L and capital trading at state H. The discussion is the same with the discussion of equations (11) and (12). Additionally, the equation (20) has an additional term $\bar{h_0} \bar{\theta_{0b}}^3 (\bar{\gamma_L}^3 - \bar{\gamma_L}^2)) \bar{\phi_0} dp_L$, compared with the equation (13). It means that the transfer of present value loss from agents 3 to 2 is also influence by dp_L. If $dp_L > 0$, the transfer is increased by a higher h_0.

The equation (21) characterizes how the increase of financial innovation cost influences social welfare. Since $\bar{\theta_{0b}}^1 < 0$, the sign of the equation (21) is negative. Since increasing h_0 requires more CPI contracts for each loan, the higher financial innovation cost reduces social welfare.

Proposition 2. *When there is default in the state L, the sign of the effect of the perturbation $dh_0 > 0$ on social welfare is determined by the sign and magnitude of the following effects and externalities:*

i) Heterogeneity effect: the sign is positive and the magnitude is determined by the difference in the present values, $\bar{\gamma_L}^i$ and the net trading positions of loan contracts, $\bar{\theta_{0b}}^3$ and the CPI return, r_L^j;

ii) Pecuniary externalities: The sign and magnitude of pecuniary externalities are determined by the following variables: the difference in the present value $\bar{\gamma_s}^i, s \in S_1$, and the net trading positions on capital $k_s^i, s \in S_1$ and loan contracts θ_{0b}^i and θ_{Lb}^i, and the sensitivity of equilibrium prices to the perturbation $dh_0 > 0$, $\frac{dm_L^b}{dh_0}, \frac{dp_s}{dh_0}, s \in S_1$,

iii)Financial innovation cost effect: the sign is negative and the magnitude is determined by the net trading positions of loan contracts, $\bar{\theta_{0b}}^1$.

Proposition 2 shows the information needed to determine whether increasing the collateral requirement on CPI contracts increases social welfare in the CPI-economy. The heterogeneous effect is a positive force on welfare improvement. It arises because agents 2 and 3 hold heterogeneous beliefs. Trading in the CPI market between agents 2 and 3, whereas eliminates heterogeneity effects caused by the belief disagreement between agents 1 and the others. The size of the welfare improvement is determined by two factors: i) the transfer of the present value loss from agents 3 to 2; ii) the return of CPI contracts in the state L, $r_L^j = 1 - \bar{\phi_0} \bar{p_L}$.

The perturbation $dh_0 > 0$ induces no collateral effect. Since $d\phi_0 = 0$, the tightness of collateral constraint on capital remains the same, $\bar{\mu}^1 \times d\phi = 0$. Also, this CPI market clearing ensures that increasing h will not induce the shortage of collateral. Hence, $\bar{\eta}^1$ does not affect social welfare.

Pecuniary externalities arise for two reasons: (i) agents hold heterogeneous beliefs; (ii) the perturbation $dh_0 > 0$ also influences capital prices and the funding cost at date 1. Since CPI contracts substitute capital goods to serve as collateral, increasing h_0 may lead capital prices to decrease, resulting in a decline of social welfare. If agents share similar beliefs, negative pecuniary externalities are stronger than heterogeneity effects. Introducing CPI contracts may reduce social welfare.

With the fundamental values in the table 3, when belief disagreement is 0.15 or 0.25, the heterogeneity effect and the pecuniary externalities are shown in the following table. Comparing the sizes of the following effects or pecuniary externalities, I find that tables 22 and 23 show that heterogeneity effects are dominant, while pecuniary externalities are close to zero.

Table 22: The effect of perturbations, dh, when $\sigma = 0.15$

$\mathit{\Gamma}_L^{32}$	$P_L^{h,b}$	$P_H^{h.k}$	$P_L^{h.k}$	$\mathit{\Upsilon}_0$
0.0006	0	0	0	0

Table 23: The effect of perturbations, dh, when $\sigma = 0.25$

$\mathit{\Gamma}_L^{32}$	$P_L^{h,b}$	$P_H^{h.k}$	$P_L^{h.k}$	$\mathit{\Upsilon}_0$
0.0022	0	0	-0.0001	0

The financial innovation cost effect arises because CPI sellers need to collect information of medium and small firms. As economies of scale cuts the financial innovation cost, the effect is trivial if the size of financial intermediaries is big.

5 Conclusion

This paper analyzes how belief heterogeneity influences the leverage cycle and the effectiveness of macro-prudential policies. The central feature of the model is that agents lend by buying risky collateralized loans from borrowers who do not share the same beliefs. Lenders might lose if borrowers default. Macro-prudential policies reduce lenders' loss by requiring borrowers to hold more collateral. The interventions affect loan trades and the volatility of leverage across the cycle. The simulation results show that a decrease in capital price occurs with a decrease in leverage. Increasing the collateral requirement on capital reduces the volatility of leverage. However, the lower volatility of leverage may not induce welfare-improving. Additionally, belief heterogeneity influences the decline of capital prices from normal times to crisis times. When belief heterogeneity is under the moderate level, capital prices in the crisis times will not induce default. The policy takeaway is that policymakers need to identify and measure belief heterogeneity to design macro-prudential policies.

Increasing the collateral requirement on capital induces three types of effects on social welfare. Heterogeneous effects increase social welfare when the macro-prudential perturbation benefits agents who value their utilities in the bust state more. Collateral effects reduce social welfare when the higher collateral requirement increases the funding cost of borrowers. Pecuniary externalities occur when the heterogeneous agents value the asset returns differently. The sign of pecuniary externalities depends on capital price changes. The sign of the effect of the perturbation on social welfare is determined by the sign and magnitude of these effects and externalities. I propose an alternative macro-prudential policy tool, namely collateral hedging, which can both reduce lenders' loss and stimulate economic activities. The simulation results show that introducing CPI contracts increases the volatility of leverage across the cycle. Although pecuniary externalities reduce social welfare, this perturbation may increase social welfare. The reason is that increasing collateral requirements on CPI contracts induce heterogeneity effects and no collateral effects.

This paper also implies that financial intermediaries play an important role in implementing the macro-prudential policies with financial innovation. Selling CPI contracts requires the ability to evaluate risk and the certification of issuing insurance contracts. Even though governments have the same advantages for providing CPI contracts, political and interest group resistance, and weakness in the governance framework influence policy stances (Bengtsson, 2019; Claessens, 2015). CPI trading eliminates the political economy constraints. The future work could focus on introducing financial innovation as an alternative policy

Appendix

A. General equilibrium in the L-economy

A collateral general equilibrium is a collection of prices, commodity holdings and contract trades $((\bar{m_0^b}, \bar{m_L^b}, \bar{p_H}, \bar{p_L})$, $(\bar{c_s}^i, \bar{k_0}, \bar{k_L}^i, \bar{k_H}^i \bar{\theta_{0b}}^i, \bar{\theta_{Lb}}^i))$ such that solves maximum problems as follow and market clearing conditions.

Maximum problem

Type 1

$$max ln(c_0^1) + \beta(\pi_H^1 ln(c_H^1) + \pi_L^1 ln(c_L^1)) + \beta^2((\pi_{HH}^1 ln(c_{HH}^1) + \pi_{HL}^1 ln(c_{HL}^1)) + \beta^2((\pi_{LL}^1 ln(c_{LL}^1) + \pi_{LH}^1 ln(c_{LH}^1))$$

$$\text{subject to, } (c_0^1 + \alpha k_0 - e_0^1) + m_0^b \theta_{0b}^1 = 0,$$

$$c_H^1 + m_H^b \theta_{Hb}^1 + p_H k_H^1 = e_H^1 + p_H k_0 + \theta_{0b}^1 \times min\{1, \phi_0 p_H\},$$

$$c_L^1 + m_L^b \theta_{Lb}^1 + p_L k_L^1 = e_L^1 + p_L k_0 + \theta_{0b}^1 \times min\{1, \phi_0 p_L\},$$

$$c_{HH}^1 = e_{HH}^1 + k_H^1 r_{HH}^k,$$

$$c_{HL}^1 = e_{HL}^1 + k_H^1 r_{HL}^k,$$

$$c_{LH}^1 = e_{LH}^1 + k_L^1 r_{LH}^k + \theta_{Lb}^1 \times min\{1, \phi_L r_{LH}^k\},$$

$$c_{LL}^1 = e_{LL}^1 + k_L^1 r_{LL}^k + \theta_{Lb}^1 \times min\{1, \phi_L r_{LL}^k\},$$

$$-\theta_{0b}^1 \phi_0 = k_0,$$

$$-\theta_{Lb}^1 \phi_L = k_L.$$

The first budget constraint requires that money spent on consumption goods beyond the revenue from endowments and production in state 0 be financed out of the sale of contracts. The second and third budget constraints require money spent on the consumption goods and capital goods beyond the revenue from endowments and production in any state $s \in S_1$ be financed out of net revenue from return from contracts bought or sold in state 0. From the fourth The last constraint requires that agents actually hold as least as much of capital as financial contracts require them to hold.

Suppose $\lambda_s^1$are the Lagrangian multipliers for the budget constraints, and μ_0^1 and μ_L^1 are the Lagrangian multipliers for the the collateral constraints on capital goods.

In the equilibrium, for each agents i, $\bar{\lambda_s}^i = \partial u^i(\bar{c_s}^i)/\partial c_s^i, s \in S$. Also, I assume the present value of agents is $\bar{\gamma_{s^*}}^i = \bar{\lambda_{s^*}}^i/\bar{\lambda_0}^i$ and $\bar{\gamma_{s_T}}^i = \bar{\lambda_{s_T}}^i/\bar{\lambda_{s^*}}^i$. $\bar{\lambda_s}^i$ and $\bar{\gamma_s}^i$ will be used in the following appendix.

Assume that $\bar{p_L} < 1/\bar{\phi_0}$, $\bar{p_H} \geq 1/\bar{\phi_0}$.

The Euler equations are

$$\theta_{0b}^1 : \bar{m_0^b} = \bar{\gamma_L}^1\bar{\phi_0}\bar{p_L} + \bar{\gamma_H}^1 + \frac{\bar{\phi_0}\bar{\mu_0}^1}{\bar{\lambda_0}^1}, \tag{A.1}$$

$$k_0 : \alpha = \bar{\gamma_L}^1\bar{p_L} + \bar{\gamma_H}^1\bar{p_H} + \frac{\bar{\mu}_0^1}{\bar{\lambda_0}^1}, \tag{A.2}$$

$$\theta_{Lb}^1 : \bar{m_L^b} = \bar{\gamma_{LL}}^1\bar{\phi_L}r_{LL}^k + \bar{\gamma_{LH}}^1 + \frac{\bar{\phi_L}\bar{\mu_L}^1}{\bar{\lambda_L}^1}, \tag{A.3}$$

$$k_L^1 : \bar{p_L} = \bar{\gamma_{LL}}^1 r_{LL}^k + \bar{\gamma_{LH}}^1 r_{LH}^k + \frac{\bar{\mu}_L^1}{\bar{\lambda_L}^1}, \tag{A.4}$$

$$k_H^1 : \bar{p_H} = \bar{\gamma_{HL}}^1 r_{HL}^k + \bar{\gamma_{HH}}^1 r_{HH}^k. \tag{A.5}$$

Type 2

$$max ln(c_0^2)+\beta(\pi_H^2 ln(c_H^2)+\pi_L^2 ln(c_L^2))+\beta^2((\pi_{HH}^2 ln(c_{HH}^2)+\pi_{HL}^2 ln(c_{HL}^2))+\beta^2((\pi_{LL}^2 ln(c_{LL}^2)+\pi_{LH}^2 ln(c_{LH}^2))$$

$$\text{subject to, } (c_0^2 - e_0^2) + m_0^b\theta_{0b}^2 = 0,$$

$$c_H^2 + m_H^b\theta_{Hb}^2 + p_H k_H^2 = e_H^2 + \theta_{0b}^2 \times min\{1, \phi_0 p_H\},$$

$$c_L^2 + m_L^b\theta_{Lb}^2 + p_L k_L^2 = e_L^2 + \theta_{0b}^2 \times min\{1, \phi_0 p_L\},$$

$$c_{HH}^2 = e_{HH}^2 + k_H^2 r_{HH}^k,$$

$$c_{HL}^2 = e_{HL}^2 + k_H^2 r_{HL}^k,$$

$$c_{LH}^2 = e_{LH}^2 + k_L^2 r_{LH}^k + \theta_{Lb}^2 \times min\{1, \phi_L r_{LH}^k\},$$

$$c_{LL}^2 = e_{LL}^2 + k_L^2 r_{LL}^k + \theta_{Lb}^2 \times min\{1, \phi_L r_{LL}^k\}.$$

Hence, the Euler equations are

$$\theta_{0b}^2 : \bar{m_0^b} = \bar{\gamma_L}^2\bar{\phi_0}\bar{p_L} + \bar{\gamma_H}^2, \tag{A.6}$$

$$\theta_{Lb}^2 : \bar{m_L^b} = \bar{\gamma_{LL}}^2\bar{\phi_L}r_{LL}^k + \bar{\gamma_{LH}}^2, \tag{A.7}$$

$$k_L^2 : \bar{p_L} = \bar{\gamma_{LL}}^2 r_{LL}^k + \bar{\gamma_{LH}}^2 r_{LH}^k, \tag{A.8}$$

$$k_H^2 : \bar{p_H} = \gamma \bar{_{HL}}^2 r^k_{HL} + \gamma \bar{_{HH}}^2 r^k_{HH}. \tag{A.9}$$

Market clearing conditions:

$$-w^1 \bar{\theta_{0b}}^1 = w^2 \bar{\theta_{0b}}^2 + w^3 \bar{\theta_{0b}}^3$$

$$-w^1 \bar{\theta_{Lb}}^1 = w^2 \bar{\theta_{Lb}}^2 + w^3 \bar{\theta_{Lb}}^3$$

$$w^1 \bar{k_L}^1 + w^2 \bar{k_L}^2 + w^3 \bar{k_L}^3 = w^1 \bar{k_0}$$

$$w^1 \bar{k_H}^1 + w^2 \bar{k_H}^2 + w^3 \bar{k_H}^3 = w^1 \bar{k_0}$$

Type 3

$$max ln(c_0^3) + \beta(\pi_H^3 ln(c_H^3) + \pi_L^3 ln(c_L^3)) + \beta^2((\pi_{HH}^3 ln(c_{HH}^3) + \pi_{HL}^3 ln(c_{HL}^3)) + \beta^2((\pi_{LL}^3 ln(c_{LL}^3) + \pi_{LH}^3 ln(c_{LH}^3))$$

$$\text{subject to, } (c_0^3 - e_0^3) + m_0^b \theta_{0b}^3 = 0,$$

$$c_H^3 + m_H^b \theta_{Hb}^3 + p_H k_H^3 = e_H^3 + \theta_{0b}^3 \times min\{1, \phi_0 p_H\},$$

$$c_L^3 + m_L^b \theta_{Lb}^3 + p_L k_L^3 = e_L^3 + p_L k_0 + \theta_{0b}^3 \times min\{1, \phi_0 p_L\},$$

$$c_{HH}^3 = e_{HH}^3 + k_H^3 r^k_{HH},$$

$$c_{HL}^3 = e_{HL}^3 + k_H^3 r^k_{HL},$$

$$c_{LH}^3 = e_{LH}^3 + k_L^3 r^k_{LH} + \theta_{Lb}^3 \times min\{1, \phi_L r^k_{LH}\},$$

$$c_{LL}^3 = e_{LL}^3 + k_L^3 r^k_{LL} + \theta_{Lb}^3 \times min\{1, \phi_L r^k_{LL}\},$$

Hence, the Euler equations are,

$$\theta_{0b}^3 : \bar{m_0^b} = \bar{\gamma_L}^3 \bar{\phi_0} \bar{p_L} + \bar{\gamma_H}^3, \tag{A.10}$$

$$\theta_{Lb}^3 : \bar{m_L^3} = \gamma \bar{_{LL}}^3 \bar{\phi_L} r^k_{LL} + \gamma \bar{_{LH}}^3, \tag{A.11}$$

$$k_L^3 : \bar{p_L} = \gamma \bar{_{LL}}^3 r^k_{LL} + \gamma \bar{_{LH}}^3 r^k_{LH}, \tag{A.12}$$

$$k_H^3 : \bar{p_H} = \gamma \bar{_{HL}}^3 r^k_{HL} + \gamma \bar{_{HH}}^3 r^k_{HH}. \tag{A.13}$$

Market clearing conditions:

$$-w^1 \bar{\theta_{0b}}^1 = w^2 \bar{\theta_{0b}}^2 + w^3 \bar{\theta_{0b}}^3$$

$$-w^1\bar{\theta_{Lb}}^1 = w^2\bar{\theta_{Lb}}^2 + w^3\bar{\theta_{Lb}}^3$$

$$w^1\bar{k_L}^1 + w^2\bar{k_L}^2 + w^3\bar{k_L}^3 = w^1\bar{k_0}$$

$$w^1\bar{k_H}^1 + w^2\bar{k_H}^2 + w^3\bar{k_H}^3 = w^1\bar{k_0}$$

The general equilibrium $((\bar{m_0^b}, \bar{m_L^b}, \bar{p_H}, \bar{p_L}), (\bar{c_s}^i, \bar{k_0}, \bar{k_L}^i, \bar{k_H}^i, \bar{\theta_{0b}}^i, \bar{\theta_{Lb}}^i))$ should satisfy budget constraints, four Euler equations, and market clearing conditions. If the results $\bar{p_s}$ satisfy the assumption that $\bar{p_L} < 1/\bar{\phi_0}$, $\bar{p_H} \geq 1/\bar{\phi_0}$, the $((\bar{m_0^b}, \bar{m_L^b}, \bar{p_H}, \bar{p_L}), (\bar{c_s}^i, \bar{k_0}, \bar{k_L}^i, \bar{k_H}^i\bar{\theta_{0b}}^i, \bar{\theta_{Lb}}^i))$ is the solution of this optimization problem in the default case.

If $\bar{p_L} \geq 1/\bar{\phi_0}$, $\bar{p_H} \geq 1/\bar{\phi_0}$, the Euler equations (A.1), (A.6) and (A.10) are changed into:

$$\theta_{0b}^1 : \bar{m_0^b} = \bar{\gamma_L}^1 + \bar{\gamma_H}^1 + \frac{\bar{\phi_0}\bar{\mu_0}^1}{\bar{\lambda_0}^1}, \tag{A.14}$$

$$\theta_{0b}^2 : \bar{m_0^b} = \bar{\gamma_L}^2 + \bar{\gamma_H}^2, \tag{A.15}$$

$$\theta_{0b}^3 : \bar{m_0^b} = \bar{\gamma_L}^3 + \bar{\gamma_H}^3. \tag{A.16}$$

B. General equilibrium in the CPI-economy

A collateral equilibrium is a collection of prices, commodity holdings and contract trades

$((\bar{m_0^b}, \bar{m_L^b}, \bar{m_0^j}, \bar{p_H}, \bar{p_L}), (\bar{c_s}^i, \bar{k_0}, \bar{k_L}^i, \bar{k_H}^i\bar{\theta_{0b}}^i, \bar{\theta_{Lb}}^i, \bar{\theta_{0j}}^i))$ such that solves maximum problems as follow and market clearing conditions.

Maximum problem

Type 1

$$max ln(c_0^1)+\beta(\pi_H^1 ln(c_H^1)+\pi_L^1 ln(c_L^1))+\beta^2((\pi_{HH}^1 ln(c_{HH}^1)+\pi_{HL}^1 ln(c_{HL}^1))+\beta^2((\pi_{LL}^1 ln(c_{LL}^1)+\pi_{LH}^1 ln(c_{LH}^1))$$

$$\text{subject to, } (c_0^1 + \alpha k_0 - e_0^1) + m_0^b\theta_{0b}^1 + m_0^j\theta_{0j}^1 = 0,$$

$$c_H^1 + m_H^b\theta_{Hb}^1 + p_H k_H^1 = e_H^1 + p_H k_0 + \theta_{0j}^1 r_H^j + \theta_{0b}^1 \times min\{1, \phi_0 p_H + h_0 r_H^j\},$$

$$c_L^1 + m_L^b\theta_{Lb}^1 + p_L k_L^1 = e_L^1 + p_L k_0 + \theta_{0j}^1 r_L^j + \theta_{0b}^1 \times min\{1, \phi_0 p_L + h_0 r_L^j\},$$

$$c_{HH}^1 = e_{HH}^1 + k_H^1 r_{HH}^k,$$

$$c_{HL}^1 = e_{HL}^1 + k_H^1 r_{HL}^k,$$

$$c_{LH}^1 = e_{LH}^1 + k_L^1 r_{LH}^k + \theta_{Lb}^1 \times min\{1, \phi_L r_{LH}^k\},$$

$$c_{LL}^1 = e_{LL}^1 + k_L^1 r_{LL}^k + \theta_{Lb}^1 \times min\{1, \phi_L r_{LL}^k\},$$

$$-\theta^1_{0b}\phi_0 = k_0,$$

$$-\theta^1_{0b}h_0 = \theta^1_{0j},$$

$$-\theta^1_{Lb}\phi_L = k_L.$$

The first seven constraints are budget constraints, and the last three constraints are collateral constraints.

Suppose η^1_0 is the Lagrangian multiplier for the the collateral constraint on CPIs.

Assume that $\bar{p_L} < 1/\bar{\phi_0}$, $\bar{p_H} \geq 1/\bar{\phi_0}$.

The Euler equations are:

$$\theta^1_{0b} : \bar{m^b_0} = \bar{\gamma_L}^1(\bar{\phi_0}\bar{p_L} + \bar{h_0}r^j_L) + \bar{\gamma_H}^1 + \frac{\bar{\phi_0}\bar{\mu}^1_0 + \bar{h_0}\bar{\eta_0}^1}{\bar{\lambda_0}^1}, \tag{B.1}$$

$$\theta^1_{0j} : \bar{m^j_0} = \bar{\gamma_L}^1 r^j_L + \frac{\bar{\eta_0}^1}{\bar{\lambda_0}^1}, \tag{B.2}$$

$$k_0 : \alpha = \bar{\gamma_L}^1\bar{p_L} + \bar{\gamma_H}^1\bar{p_H} + \frac{\bar{\mu}^1_0}{\bar{\lambda_0}^1}, \tag{B.3}$$

$$\theta^1_{Lb} : \bar{m^b_L} = \bar{\gamma_{LL}}^1\bar{\phi_L}r^k_{LL} + \bar{\gamma_{LH}}^1 + \frac{\bar{\phi_L}\bar{\mu_L}^1}{\bar{\lambda_L}^1}, \tag{B.4}$$

$$k^1_L : \bar{p_L} = \bar{\gamma_{LL}}^1 r^k_{LL} + \bar{\gamma_{LH}}^1 r^k_{LH} + \frac{\bar{\mu}^1_L}{\bar{\lambda_L}^1}, \tag{B.5}$$

$$k^1_H : \bar{p_H} = \bar{\gamma_{HL}}^1 r^k_{HL} + \bar{\gamma_{HH}}^1 r^k_{HH}. \tag{B.6}$$

Type 2

$$max ln(c^2_0)+\beta(\pi^2_H ln(c^2_H)+\pi^2_L ln(c^2_L))+\beta^2((\pi^2_{HH} ln(c^2_{HH})+\pi^2_{HL} ln(c^2_{HL}))+\beta^2((\pi^2_{LL} ln(c^2_{LL})+\pi^2_{LH} ln(c^2_{LH}))$$

$$\text{subject to, } (c^2_0 - e^2_0) + m^b_0\theta^2_{0b} + m^j_0\theta^2_{0j} - \varepsilon\theta^2_{0j} = 0,$$

$$c^2_H + m^b_H\theta^2_{Hb} + p_H k^2_H = e^2_H + \theta^2_{0j}r^j_H + \theta^2_{0b} \times min\{1, \phi_0 p_H + h_0 r^j_H\},$$

$$c^2_L + m^b_L\theta^2_{Lb} + p_L k^2_L = e^2_L + p_L k_0 + \theta^2_{0j}r^j_L + \theta^2_{0b} \times min\{1, \phi_0 p_L + h_0 r^j_L\},$$

$$c^2_{HH} = e^2_{HH} + k^2_H r^k_{HH},$$

$$c^2_{HL} = e^2_{HL} + k^2_H r^k_{HL},$$

$$c^2_{LH} = e^2_{LH} + k^2_L r^k_{LH} + \theta^2_{Lb} \times min\{1, \phi_L r^k_{LH}\},$$

$$c^2_{LL} = e^2_{LL} + k^2_L r^k_{LL} + \theta^2_{Lb} \times min\{1, \phi_L r^k_{LL}\},$$

Hence, the Euler equations are

$$\theta^2_{0b} : \bar{m^b_0} = \bar{\gamma_L}^2(\bar{\phi_0}\bar{p_L} + \bar{h_0}r^j_L) + \bar{\gamma_H}^2, \tag{B.7}$$

$$\theta_{0j}^2 : \bar{m_0^j} = \bar{\gamma_L}^2 r_L^j + \varepsilon, \tag{B.8}$$

$$\theta_{Lb}^2 : \bar{m_L^b} = \bar{\gamma_{LL}}^2 \bar{\phi_L} r_{LL}^k + \bar{\gamma_{LH}}^2, \tag{B.9}$$

$$k_L^2 : \bar{p_L} = \bar{\gamma_{LL}}^2 r_{LL}^k + \bar{\gamma_{LH}}^2 r_{LH}^k, \tag{B.10}$$

$$k_H^2 : \bar{p_H} = \bar{\gamma_{HL}}^2 r_{HL}^k + \bar{\gamma_{HH}}^2 r_{HH}^k. \tag{B.11}$$

Type 3

$$max ln(c_0^3)+\beta(\pi_H^3 ln(c_H^3)+\pi_L^3 ln(c_L^3))+\beta^2((\pi_{HH}^3 ln(c_{HH}^3)+\pi_{HL}^3 ln(c_{HL}^3))+\beta^2((\pi_{LL}^3 ln(c_{LL}^3)+\pi_{LH}^3 ln(c_{LH}^3))$$

$$\text{subject to, } (c_0^3 - e_0^3) + m_0^b \theta_{0b}^3 = 0,$$

$$c_H^3 + m_H^b \theta_{Hb}^3 + p_H k_H^3 = e_H^3 + \theta_{0b}^3 \times min\{1, \phi_0 p_H + h_0 r_H^j\},$$

$$c_L^3 + m_L^b \theta_{Lb}^3 + p_L k_L^3 = e_L^3 + p_L k_0 + \theta_{0b}^3 \times min\{1, \phi_0 p_L + h_0 r_L^j\},$$

$$c_{HH}^3 = e_{HH}^3 + k_H^3 r_{HH}^k,$$

$$c_{HL}^3 = e_{HL}^3 + k_H^3 r_{HL}^k,$$

$$c_{LH}^3 = e_{LH}^3 + k_L^3 r_{LH}^k + \theta_{Lb}^3 \times min\{1, \phi_L r_{LH}^k\},$$

$$c_{LL}^3 = e_{LL}^3 + k_L^3 r_{LL}^k + \theta_{Lb}^3 \times min\{1, \phi_L r_{LL}^k\},$$

Hence, the Euler equations are

$$\theta_{0b}^3 : \bar{m_0^b} = \bar{\gamma_L}^3 (\bar{\phi_0} \bar{p_L} + \bar{h_0} r_L^j) + \bar{\gamma_H}^3, \tag{B.12}$$

$$\theta_{Lb}^3 : \bar{m_L^3} = \bar{\gamma_{LL}}^3 \bar{\phi_L} r_{LL}^k + \bar{\gamma_{LH}}^3, \tag{B.13}$$

$$k_L^3 : \bar{p_L} = \bar{\gamma_{LL}}^3 r_{LL}^k + \bar{\gamma_{LH}}^3 r_{LH}^k, \tag{B.14}$$

$$k_H^3 : \bar{p_H} = \bar{\gamma_{HL}}^3 r_{HL}^k + \bar{\gamma_{HH}}^3 r_{HH}^k. \tag{B.15}$$

Market clearing conditions:

$$-w^1 \bar{\theta_{0b}}^1 = w^2 \bar{\theta_{0b}}^2 + w^3 \bar{\theta_{0b}}^3$$

$$-w^1 \bar{\theta_{Lb}}^1 = w^2 \bar{\theta_{Lb}}^2 + w^3 \bar{\theta_{Lb}}^3$$

$$-w^1\bar{\theta_{0j}}^1 = w^2\bar{\theta_{0j}}^2$$

$$w^1\bar{k_L}^1 + w^2\bar{k_L}^2 + w^3\bar{k_L}^3 = w^1\bar{k_0}$$

$$w^1\bar{k_H}^1 + w^2\bar{k_H}^2 + w^3\bar{k_H}^3 = w^1\bar{k_0}$$

The general equilibrium $((\bar{m_0^b}, \bar{m_L^b}, \bar{m_0^j}, \bar{p_H}, \bar{p_L}), (\bar{c_s}^i, \bar{k_0}, \bar{k_L}^i, \bar{k_H}^i \bar{\theta_{0b}}^i, \bar{\theta_{Lb}}^i, \bar{\theta_{0j}}^i))$ should satisfy budget constraints, six Euler equations and market clearing conditions. If the results $\bar{p_s}$ satisfy the assumption that $\bar{p_L} < 1/\bar{\phi_0}$, $\bar{p_H} \geq 1/\bar{\phi_0}$, the $((\bar{m_0^b}, \bar{m_L^b}, \bar{m_0^j}, \bar{p_H}, \bar{p_L}), (\bar{c_s}^i, \bar{k_0}, \bar{k_L}^i, \bar{k_H}^i \bar{\theta_{0b}}^i, \bar{\theta_{Lb}}^i, \bar{\theta_{0j}}^i))$ is the solution of this optimization problem in the default case.

C. Proofs

I. Macro-prudential perturbations in the L-economy

In the L-economy, the perturbation $d\phi_0$ at the initial state, where $d\phi_0 > 0$ induce marginal changes at date 0, $(dc_0^i, dk_0, d\theta_{0b}^i)$,with $\sum_{i=1}^{3} w^i d\theta_{0b}^i = 0$, and then adjustments of the subsequent equilibrium plans and prices $(dc_s^i, dk_L^i, dk_H^i, , dm_0^b, dm_L^b, dp_L, dp_H)$ around the equilibrium $(\bar{c_s}^i, \bar{k_0}, \bar{k_L}^i, \bar{k_H}^i, \bar{p_H}, \bar{p_L}, \bar{\theta_{0b}}^i, \bar{\theta_{Lb}}^i, \bar{m_0^b}, \bar{m_L^b})$.

Then, I compute the marginal change of consumption distribution of each type of agents, relative to the stationary competitive equilibrium, following a marginal change of the policy parameter $d\phi_0$. Because of market clearing conditions, the effect on social welfare does not require compute dm_0^b. Then, I compute the marginal changes of utilities of each type of agents and social welfare.

The change in agents i's marginal utility is given by

$$\frac{du^i}{\bar{\lambda_0}^i} \mid_{\phi_0=\bar{\phi_0}} = dc_0^i + \bar{\gamma_L}^i(dc_L^i + \bar{\gamma_{LL}}^i dc_{LL}^i + \bar{\gamma_{LH}}^i dc_{LH}^i) + \bar{\gamma_H}^i(dc_H^i + \bar{\gamma_{HL}}^i dc_{HL}^i + \bar{\gamma_{HH}}^i dc_{HH}^i). \qquad \text{(C.1)}$$

Case 1: **If in the equilibrium, $\bar{p_L} < 1/\bar{\phi}$, $\bar{p_H} \geq 1/\bar{\phi}$, there is default in the state L.**

Type 1

The change of type 1 consumption at date 0 is

$$dc_0^1 \mid_{\phi_0=\bar{\phi_0}} = -\bar{\theta_{0b}}^1 dm_0^b - \bar{m_0^b} d\theta_{0b}^1 - a dk_0. \qquad \text{(C.2)}$$

Then, substitute (A.1), (A.2) into (C.2), which are from first order conditions for an optimum at the stationary competitive equilibrium:

$$\begin{aligned} dc_0^1 \mid_{\phi_0=\bar{\phi_0}} = & -\bar{\theta_{0b}}^1 dm_0^b - (\bar{\gamma_L}^1 \bar{\phi_0}\bar{p_L} + \bar{\gamma_H}^1 + \frac{\bar{\phi_0}\bar{\mu_0^1}}{\bar{\lambda_0}^1}) d\theta_{0b}^1 - \\ & - (\bar{\gamma_L}^1 \bar{p_L} + \bar{\gamma_H}^1 \bar{p_H} + \frac{\bar{\mu_0}^1}{\bar{\lambda_0}^1}) dk_0. \end{aligned}$$

Since both collateral constraint bind, $k_0 = -\phi_0 \theta_{0b}^1$. Hence, the marginal changes $dk_0 = -\bar{\phi}_0 d\theta_{0b}^1 - \bar{\theta}_{0b}^{1} d\phi_0$, which yields,

$$\begin{aligned} dc_0^1 \mid_{\phi_0=\bar{\phi}_0} = & - \bar{\theta}_{0b}^{1} dm_0^b - (\bar{\gamma}_L^{1} \bar{\phi}_0 \bar{p}_L + \bar{\gamma}_H^{1} + \frac{\bar{\phi}_0 \bar{\mu}_0^{1}}{\bar{\lambda}_0^{1}}) d\theta_{0b}^1 - \\ & (\bar{\gamma}_L^{1} \bar{p}_L + \bar{\gamma}_H^{1} \bar{p}_H + \frac{\bar{\mu}_0^{1}}{\bar{\lambda}_0^{1}})(-\bar{\phi}_0 d\theta_{0b}^1 - \bar{\theta}_{0b}^{1} d\phi_0) \end{aligned}$$

After simplifying, I obtain

$$dc_0^1 \mid_{\phi_0=\bar{\phi}_0} = -\bar{\theta}_{0b}^{1} dm_0^b + \bar{\phi}_0 \bar{\gamma}_H^{1} \bar{p}_H d\theta_{0b}^1 - \bar{\gamma}_H^{1} d\theta_{0b}^1 + (\bar{\gamma}_L^{1} \bar{p}_L + \bar{\gamma}_H^{1} \bar{p}_H + \frac{\bar{\mu}_0^{1}}{\bar{\lambda}_0^{1}}) \bar{\theta}_{0b}^{1} d\phi_0 \tag{C.3}$$

The change of type 1 consumption in the L state is

$$dc_L^1 \mid_{\phi_0=\bar{\phi}_0} = -\bar{\theta}_{Lb}^{1} dm_L^b - \bar{m}_L^b d\theta_{Lb}^1 - \bar{p}_L dk_L^1 - \bar{k}_L^{1} dp_L + \bar{\phi}_0 \bar{p}_L d\theta_{0b}^1 + \bar{\theta}_{0b}^{1} \bar{p}_L d\phi_0 + \bar{\phi}_0 \bar{\theta}_{0b}^{1} dp_L + \bar{p}_L dk_0 + \bar{k}_0 dp_L.$$

Since collateral constraints require $dk_0 = -\bar{\phi}_0 d\theta_b^1 - \bar{\theta}_b^{1} d\phi_0$, I obtain

$$\begin{aligned} dc_L^1 \mid_{\phi_0=\bar{\phi}_0} = & - \bar{\theta}_{Lb}^{1} dm_L^b - \bar{m}_L^b d\theta_{Lb}^1 - \bar{p}_L dk_L^1 - \bar{k}_L^{1} dp_L + \bar{\phi}_0 \bar{p}_L d\theta_{0b}^1 \\ & + \bar{\theta}_{0b}^{1} \bar{p}_L d\phi_0 + \bar{p}_L(-\bar{\phi}_0 d\theta_{0b}^1 - \bar{\theta}_{0b}^{1} d\phi_0) + (\bar{\phi}_0 \bar{\theta}_{0b}^{1} + \bar{k}_0) dp_L \\ = & - \bar{\theta}_{Lb}^{1} dm_L^b - \bar{m}_L^b d\theta_{Lb}^1 - \bar{p}_L dk_L^1 - \bar{k}_L^{1} dp_L. \end{aligned} \tag{C.4}$$

Then, substitute (A.3), (A.4) into (C.4), which are from first order conditions for an optimum at the stationary competitive equilibrium:

$$\begin{aligned} dc_L^1 \mid_{\phi_0=\bar{\phi}_0} = & - \bar{\theta}_{Lb}^{1} dm_L^b - \bar{m}_L^b d\theta_{Lb}^1 - \bar{p}_L dk_L^1 - \bar{k}_L^{1} dp_L \\ = & - \bar{\theta}_{Lb}^{1} dm_L^b - (\bar{\gamma}_{LL}^{1} \bar{\phi}_L r_{LL}^k + \bar{\gamma}_{LH}^{1} + \frac{\bar{\phi}_L \bar{\mu}_L^{1}}{\bar{\lambda}_L^{1}}) d\theta_{Lb}^1 - (\bar{\gamma}_{LL}^{1} r_{LL}^k + \bar{\gamma}_{LH}^{1} r_{LH}^k + \frac{\bar{\mu}_L^1}{\bar{\lambda}_L^{1}}) dk_L^1 - \bar{k}_L^{1} dp_L \\ = & - \bar{\theta}_{Lb}^{1} dm_L^b - \bar{\gamma}_{LH}^{1} d\theta_{Lb}^1 - \bar{\gamma}_{LH}^{1} r_{LH}^k dk_L^1 - \bar{k}_L^{1} dp_L \end{aligned} \tag{C.5}$$

The change of type 1 consumption in the H state is

$$dc_H^1 \mid_{\phi_0=\bar{\phi}_0} = -\bar{p}_H dk_H^1 - \bar{k}_H^{1} dp_H + d\theta_{0b}^1 + \bar{p}_H dk_0 + \bar{k}_0 dp_H. \tag{C.6}$$

Then, substitute (A.1) into (C.6), which are from first order conditions for an optimum at the stationary competitive equilibrium:

$$dc_H^1 \mid_{\phi_0=\bar{\phi}_0} = -\bar{k}_H^{1} dp_H + d\theta_{0b}^1 + (\bar{\gamma}_{HL}^{1} r_{HL}^k + \bar{\gamma}_{HH}^{1} r_{HH}^k)(dk_0 - dk_H^1) + \bar{k}_0 dp_H. \tag{C.7}$$

The change of type 1 consumption in the LH state is,

$$dc_{LH}^1 \mid_{\phi_0=\bar{\phi}_0} = d\theta_{Lb}^1 + r_{LH} dk_L^1. \tag{C.8}$$

The change of type 1 consumption in the LL state is,

$$dc_{LL}^1 \mid_{\phi_0=\bar{\phi}_0} = \bar{\phi}_L r_{LL}^k d\theta_{Lb}^1 + r_{LL} dk_L^1.$$

Since the collateral requirement at state L requires $dk_L^1 = -\bar{\phi}_L d\theta_{Lb}^1$,

$$dc^1_{LL}\mid_{\phi_0=\bar{\phi_0}}=\bar{\phi_L}r^k_{LL}d\theta^1_{Lb}+r_{LL}(-\bar{\phi}_Ld\theta^1_{Lb})=0. \tag{C.9}$$

The change of type 1 consumption in the HH state is,

$$dc^1_{HH}\mid_{\phi_0=\bar{\phi_0}}=r_{HH}dk^1_H. \tag{C.10}$$

The change of type 1 consumption in the HL state is,

$$dc^1_{HL}\mid_{\phi_0=\bar{\phi_0}}=r_{HL}dk^1_H. \tag{C.11}$$

Substitute equations (C.3), (C.5), (C.7), (C.8), (C.9), (C.10) and (C.11) into equation (C.1), and the marginal change of agent 1's utility is

$$\begin{aligned}\frac{du^1}{\bar{\lambda_0}^1}\mid_{\phi_0=\bar{\phi_0}} &= -\bar{\theta_{0b}}^1dm^b_0+(\bar{\gamma_L}^1\bar{p_L}+\frac{\bar{\mu_0}^1}{\bar{\lambda_0}^1})\bar{\theta_{0b}}^1d\phi_0\\ &+\bar{\gamma_L}^1(-\bar{\theta_{Lb}}^1dm^b_L-\bar{k_L}^1dp_L)+\bar{\gamma_H}^1(\bar{k_0}dp_H-\bar{k_H}^1dp_H).\end{aligned} \tag{C.12}$$

Type 2

The change of type 2 consumption at date 0 is

$$dc^2_0\mid_{\phi_0=\bar{\phi_0}}=-\bar{\theta_{0b}}^2dm^b_0-\bar{m}^b_0d\theta^2_{0b}. \tag{C.13}$$

Then, substitute (A.6) into (C.13), I obtain

$$dc^2_0\mid_{\phi_0=\bar{\phi_0}}=-\bar{\theta_{0b}}^2dm^b-(\bar{\gamma_L}^2\bar{\phi_0}\bar{p_L}+\bar{\gamma_H}^2)d\theta^2_{0b}. \tag{C.14}$$

The change of type 2 consumption in the L state is

$$dc^2_L\mid_{\phi_0=\bar{\phi_0}}=-\bar{\theta_{Lb}}^2dm^b_L-\bar{m}^b_Ld\theta^2_{Lb}-\bar{p_L}dk^2_L-\bar{k_L}^2dp_L+\bar{\phi_0}\bar{p_L}d\theta^2_{0b}+\bar{\theta_{0b}}^2\bar{p_L}d\phi_0+\bar{\phi_0}\bar{\theta_{0b}}^2dp_L. \tag{C.15}$$

Then, substitute (A.7) and (A.8) into (C.15), I obtain

$$\begin{aligned}dc^2_L\mid_{\phi_0=\bar{\phi_0}}= &-\bar{\theta_{Lb}}^2dm^b_L-(\gamma\bar{_{LL}}^2\bar{\phi_L}r^k_{LL}+\gamma\bar{_{LH}}^2)d\theta^2_{Lb}-(\gamma\bar{_{LL}}^2r^k_{LL}+\gamma\bar{_{LH}}^2r^k_{LH})dk^2_L-\bar{k_L}^2dp_L\\ &+\bar{p_L}(\bar{\phi_0}d\theta^2_{0b}+\bar{\theta_{0b}}^2d\phi_0)+\bar{\phi_0}\bar{\theta_{0b}}^2dp_L\end{aligned} \tag{C.16}$$

The change of type 2 consumption in the H state is,

$$dc^2_H\mid_{\phi_0=\bar{\phi_0}}=-\bar{p_H}dk^2_H-\bar{k_H}^2dp_H+d\theta^2_{0b}. \tag{C.17}$$

Then, substitute (A.9) into (C.17), I obtain

$$dc^2_H\mid_{\phi_0=\bar{\phi_0}}=-(\gamma\bar{_{HL}}^2r^k_{HL}+\gamma\bar{_{HH}}^2r^k_{HH})dk^2_H-\bar{k_H}^2dp_H+d\theta^2_{0b}. \tag{C.18}$$

The change of type 2 consumption in the LH state is,

$$dc^2_{LH}\mid_{\phi_0=\bar{\phi_0}}=d\theta^2_{Lb}+r_{LH}dk^2_L. \tag{C.19}$$

The change of type 2 consumption in the LL state is,

$$dc^2_{LL} \mid_{\phi_0=\bar{\phi_0}} = \bar{\phi}_L r^k_{LL} d\theta^2_{Lb} + r_{LL} dk^2_L. \tag{C.20}$$

The change of type 2 consumption in the HH state is,

$$dc^2_{HH} \mid_{\phi_0=\bar{\phi_0}} = r_{HH} dk^2_H. \tag{C.21}$$

The change of type 2 consumption in the HL state is,

$$dc^2_{HL} \mid_{\phi_0=\bar{\phi_0}} = r_{HL} dk^2_H. \tag{C.22}$$

Substitute equations (C.14), (C.16), (C.18), (C.19), (C.20), (C.21) and (C.22) into equation (C.1), and the marginal change of agent 2's utility is

$$\begin{aligned}\frac{du^2}{\bar{\lambda_0}^2} \mid_{\phi_0=\bar{\phi_0}} &= -\bar{\theta_{0b}}^2 dm^b_0 + \bar{\gamma_L}^2 \bar{p_L} \bar{\theta_{0b}}^2 d\phi_0 \\ &+ \bar{\gamma_L}^2(-\bar{\theta_{Lb}}^2 dm^b_L - \bar{k_L}^2 dp_L + \bar{\phi_0}\bar{\theta_{0b}}^2 dp_L) + \bar{\gamma_H}^2(-\bar{k_H}^2 dp_H).\end{aligned} \tag{C.23}$$

Type 3

The change of type 3 consumption at date 0 is

$$dc^3_0 \mid_{\phi_0=\bar{\phi_0}} = -\bar{\theta_{0b}}^3 dm^b_0 - \bar{m}^b_0 d\theta^3_{0b}. \tag{C.24}$$

Then, substitute (A.11) into (C.24), I obtain

$$dc^3_0 \mid_{\phi_0=\bar{\phi_0}} = -\bar{\theta_{0b}}^3 dm^b - (\bar{\gamma_L}^3 \bar{\phi_0} \bar{p_L} + \bar{\gamma_H}^3) d\theta^2_{0b}. \tag{C.25}$$

The change of type 3 consumption in the L state is

$$dc^3_L \mid_{\phi_0=\bar{\phi_0}} = -\bar{\theta_{Lb}}^3 dm^b_L - \bar{m}^b_L d\theta^3_{Lb} - \bar{p_L} dk^3_L - \bar{k_L}^3 dp_L + \bar{\phi_0}\bar{p_L} d\theta^3_{0b} + \bar{\theta_{0b}}^3 \bar{p_L} d\phi_0 + \bar{\phi_0}\bar{\theta_{0b}}^3 dp_L. \tag{C.26}$$

Then, substitute (A.12) and (A.13) into (C.26), I obtain

$$\begin{aligned}dc^3_L \mid_{\phi_0=\bar{\phi_0}} =& -\bar{\theta_{Lb}}^3 dm^b_L - (\gamma \bar{L}_L{}^3 \bar{\phi}_L r^k_{LL} + \gamma \bar{L}_H{}^3) d\theta^3_{Lb} - (\gamma \bar{L}_L{}^3 r^k_{LL} + \gamma \bar{L}_H{}^3 r^k_{LH}) dk^3_L - \bar{k_L}^3 dp_L \\ &+ \bar{p_L}(\bar{\phi_0} d\theta^3_{0b} + \bar{\theta_{0b}}^3 d\phi_0) + \bar{\phi_0}\bar{\theta_{0b}}^3 dp_L\end{aligned} \tag{C.27}$$

The change of type 3 consumption in the H state is,

$$dc^3_H \mid_{\phi_0=\bar{\phi_0}} = -\bar{p_H} dk^3_H - \bar{k_H}^3 dp_H + d\theta^3_{0b}. \tag{C.28}$$

Then, substitute (A.14) into (C.28), I obtain

$$dc^3_H \mid_{\phi_0=\bar{\phi_0}} = -(\gamma \bar{H}_L{}^3 r^k_{HL} + \gamma \bar{H}_H{}^3 r^k_{HH}) dk^3_H - \bar{k_H}^3 dp_H + d\theta^3_{0b}. \tag{C.29}$$

The change of type 3 consumption in the LH state is,

$$dc^3_{LH} \mid_{\phi_0=\bar{\phi_0}} = d\theta^3_{Lb} + r_{LH} dk^3_L. \tag{C.30}$$

The change of type 3 consumption in the LL state is,

$$dc^3_{LL}\mid_{\phi_0=\bar{\phi_0}}=\bar{\phi_L}r^k_{LL}d\theta^3_{Lb}+r_{LL}dk^3_L. \tag{C.31}$$

The change of type 3 consumption in the HH state is,

$$dc^3_{HH}\mid_{\phi_0=\bar{\phi_0}}=r_{HH}dk^3_H. \tag{C.32}$$

The change of type 3 consumption in the HL state is,

$$dc^3_{HL}\mid_{\phi_0=\bar{\phi_0}}=r_{HL}dk^3_H. \tag{C.33}$$

Substitute equations (C.25), (C.27), (C.29), (C.30), (C.31), (C.32) and (C.33) into equation (C.1), and the marginal change of agent 3's utility is

$$\begin{aligned}\frac{du^3}{\bar{\lambda_0}^3}\mid_{\phi_0=\bar{\phi_0}} &= -\bar{\theta_{0b}}^3dm^b_0+\bar{\gamma_L}^3\bar{p_L}\bar{\theta_{0b}}^3d\phi_0\\ &\quad+\bar{\gamma_L}^3(-\bar{\theta_{Lb}}^3dm^b_L-\bar{k_L}^3dp_L+\bar{\phi_0}\bar{\theta_{0b}}^3dp_L)+\bar{\gamma_H}^3(-\bar{k_H}^3dp_H).\end{aligned} \tag{C.34}$$

Social welfare

I add up equations (C.12), (C.23) and (C.34), and then

$$\begin{aligned}\sum_{i=1}^3 w^i\frac{du^i}{\bar{\lambda_0}^i}\mid_{\phi=\bar{\phi_0}} =&(w^1\frac{du^1}{\bar{\lambda_0}^1}+w^2\frac{du^2}{\bar{\lambda_0}^2}+w^3\frac{du^3}{\bar{\lambda_0}^3})\mid_{\phi=\bar{\phi_0}}\\
=&+w^1(-\bar{\theta_{0b}}^1dm^b_0+(\bar{\gamma_L}^1\bar{p_L}+\frac{\bar{\mu_0}^1}{\bar{\lambda_0}^1})\bar{\theta_{0b}}^1d\phi_0\\
&+(\bar{\gamma_L}^1(-\bar{\theta_{Lb}}^1dm^b_L-\bar{k_L}^1dp_L)+\bar{\gamma_H}^1(\bar{k_0}dp_H-\bar{k_H}^1dp_H))\\
&+w^2(-\bar{\theta_{0b}}^2dm^b_0+\bar{\gamma_L}^2\bar{p_L}\bar{\theta_{0b}}^2d\phi_0\\
&+(\bar{\gamma_L}^2(-\bar{\theta_{Lb}}^2dm^b_L-\bar{k_L}^2dp_L+\bar{\phi_0}\bar{\theta_{0b}}^2dp_L)+\bar{\gamma_H}^2(-\bar{k_H}^2dp_H))\\
&w^3(-\bar{\theta_{0b}}^3dm^b_0+\bar{\gamma_L}^3\bar{p_L}\bar{\theta_{0b}}^3d\phi_0\\
&+\bar{\gamma_L}^3(-\bar{\theta_{Lb}}^3dm^b_L-\bar{k_L}^3dp_L+\bar{\phi_0}\bar{\theta_{0b}}^3dp_L)+\bar{\gamma_H}^3(-\bar{k_H}^3dp_H))\\
=&-(w^1\bar{\theta_{0b}}^1+w^2\bar{\theta_{0b}}^2+w^3\bar{\theta_{0b}}^3)dm^b_0\\
&+w^1\bar{\theta_{0b}}^1(\bar{\gamma_L}^1\bar{p_L}+\frac{\bar{\mu_0}^1}{\bar{\lambda_0}^1})d\phi_0+w^2\bar{\theta_{0b}}^2\bar{\gamma_L}^2\bar{p_L}d\phi_0+w^3\bar{\theta_{0b}}^3\bar{\gamma_L}^3\bar{p_L}d\phi_0.\\
&-(w^1\bar{\gamma_L}^1\bar{\theta_{Lb}}^1+w^2\bar{\gamma_L}^2\bar{\theta_{Lb}}^2+w^3\bar{\gamma_L}^3\bar{\theta_{Lb}}^3)dm^b_L\\
&+(w^2\bar{\gamma_L}^2\bar{\phi_0}\bar{\theta_{0b}}^2+w^3\bar{\gamma_L}^3\bar{\phi_0}\bar{\theta_{0b}}^3-w^1\bar{\gamma_L}^1\bar{k_L}^1-w^2\bar{\gamma_L}^2\bar{k_L}^2-w^3\bar{\gamma_L}^3\bar{k_L}^3)dp_L\\
&+w^1\bar{\gamma_H}^1\bar{k_0}dp_H-(w^1\bar{\gamma_H}^1\bar{k_H}^1+w^2\bar{\gamma_H}^2\bar{k_H}^2+w^3\bar{\gamma_H}^3\bar{k_H}^3)dp_H\end{aligned}$$

When the loan contract market is clear, $-w^1\bar{\theta_{0b}}^1=w^2\bar{\theta_{0b}}^2+w^3\bar{\theta_{0b}}^3$. Then I obtain,

$$\sum_{i=1}^{3} w^i \frac{du^i}{\bar{\lambda_0}^i}|_{\phi_0=\bar{\phi_0}} = (w^2\bar{\theta_{0b}}^2(\bar{p_L}(\bar{\gamma_L}^2 - \bar{\gamma_L}^1) - \frac{\bar{\mu_0}^1}{\bar{\lambda_0}^1}) + w^3\bar{\theta_{0b}}^3(\bar{p_L}(\bar{\gamma_L}^3 - \bar{\gamma_L}^1) - \frac{\bar{\mu_0}^1}{\bar{\lambda_0}^1}))d\phi_0$$
$$+ \sum_{i=1}^{3} w^i\bar{\gamma_L}^i(-\bar{\theta_{Lb}}^i dm_L^b)$$
$$+ w^1\bar{\gamma_H}^1\bar{k_0}dp_H + \sum_{i=1}^{3} w^i\bar{\gamma_H}^i(-\bar{k_H}^i dp_H)$$
$$+ (w^2\bar{\gamma_L}^2\bar{\theta_{0b}}^2\bar{\phi_0} + w^3\bar{\gamma_L}^3\bar{\theta_{0b}}^3\bar{\phi_0} - \sum_{i=1}^{3} w^i\bar{\gamma_L}^i\bar{k_L}^i)dp_L. \tag{C.35}$$

With $\bar{\gamma_L}^1 < \bar{\gamma_L}^2 < \bar{\gamma_L}^3$, the second line of the equation of equation (C.35) is

$$\begin{aligned}&\sum_{i=1}^{3} w^i\bar{\gamma_L}^i(-\bar{\theta_{Lb}}^i dm_L^b)\\ =&(w^2(\bar{\gamma_L}^1 - \bar{\gamma_L}^2)\bar{\theta_{Lb}}^2 + w^3(\bar{\gamma_L}^1 - \bar{\gamma_L}^3)\bar{\theta_{Lb}}^3)dm_L^b\end{aligned}$$

The positive sign of the second line of the equation (C.35) requires $dm_L^b < 0$.

Since $\sum_{i=1}^{3} \bar{k_H}^i = \bar{k_0}$ The third line of the equation (C.35) is

$$\begin{aligned}&w^1\bar{\gamma_H}^1\bar{k_0}dp_H + \sum_{i=1}^{3} w^i\bar{\gamma_H}^i(-\bar{k_H}^i dp_H)\\ =&(w^2(\bar{\gamma_H}^1 - \bar{\gamma_H}^2)\bar{k_H}^2 + w^3(\bar{\gamma_H}^1 - \bar{\gamma_H}^3)\bar{k_H}^3)dp_H.\end{aligned}$$

Since $\bar{\gamma_H}^1 > \bar{\gamma_H}^2 > \bar{\gamma_H}^3$, the positive sign of the third line requires $dp_H > 0$.

In addition, since $\sum_{i=1}^{3} w^i\bar{k_L}^i = \bar{k_0}$ and $\bar{\gamma_L}^1 < \bar{\gamma_L}^2 < \bar{\gamma_L}^3$. If $w^2 \le w^3$, The fourth line of the equation (C.35) is

$$\begin{aligned}&(w^2\bar{\gamma_L}^2\bar{\theta_{0b}}^2\bar{\phi_0} + w^3\bar{\gamma_L}^3\bar{\theta_{0b}}^3\bar{\phi_0} - \sum_{i=1}^{3} w^i\bar{\gamma_L}^i\bar{k_L}^i)dp_L\\
=&(w^2\bar{\gamma_L}^2\bar{\theta_{0b}}^2\bar{\phi_0} + w^3\bar{\gamma_L}^3\bar{\theta_{0b}}^3\bar{\phi_0} - w^1\bar{\gamma_L}^1\bar{k_0}dp_L + w^1\bar{\gamma_L}^1\bar{k_0}dp_L - \sum_{i=1}^{3} w^i\bar{\gamma_L}^i\bar{k_L}^i)dp_L\\
=&(w^2\bar{\gamma_L}^2\bar{\theta_{0b}}^2\bar{\phi_0} + w^3\bar{\gamma_L}^3\bar{\theta_{0b}}^3\bar{\phi_0} - w^1\bar{\gamma_L}^1(\bar{\theta_{0b}}^2\bar{\phi_0} + \bar{\theta_{0b}}^3\bar{\phi_0})dp_L +\\
&(w^2(\bar{\gamma_L}^1 - \bar{\gamma_L}^2)\bar{k_L}^2 + w^3(\bar{\gamma_L}^1 - \bar{\gamma_L}^3)\bar{k_L}^3)dp_L\\
=&(w^2(\bar{\gamma_L}^2 - \bar{\gamma_L}^1)\bar{\theta_{0b}}^2\bar{\phi_0} + w^3(\bar{\gamma_L}^3 - \bar{\gamma_L}^1)\bar{\theta_{0b}}^3\bar{\phi_0})dp_L +\\
&(w^2(\bar{\gamma_L}^1 - \bar{\gamma_L}^2)\bar{k_L}^2 + w^3(\bar{\gamma_L}^1 - \bar{\gamma_L}^3)\bar{k_L}^3)dp_L\\
=&(w^2(\bar{\gamma_L}^2 - \bar{\gamma_L}^1)(\bar{\theta_{0b}}^2\bar{\phi_0} - \bar{k_L}^2) + w^3(\bar{\gamma_L}^3 - \bar{\gamma_L}^1)(\bar{\theta_{0b}}^3\bar{\phi_0} - \bar{k_L}^3))dp_L\\
>&(w^2(\bar{\gamma_L}^2 - \bar{\gamma_L}^1)(\bar{\theta_{0b}}^2\bar{\phi_0} - \bar{k_L}^2) + w^3(\bar{\gamma_L}^2 - \bar{\gamma_L}^1)(\bar{\theta_{0b}}^3\bar{\phi_0} - \bar{k_L}^3))dp_L\\
=&w^2(\bar{\gamma_L}^2 - \bar{\gamma_L}^1)(\bar{k_0} - \bar{k_L}^2 - \bar{k_L}^3)dp_L.\end{aligned}$$

Since $\bar{k_0} - \bar{k_L}^2 - \bar{k_L}^3 = \bar{k_L}^1 \ge 0$, the positive sign of the fourth line requires $dp_L > 0$ and agents 1 hold capital goods in the state L in the equilibrium.

Case 2: **If in the equilibrium, $\bar{p_L} \ge 1/\bar{\phi}$, $\bar{p_H} \ge 1/\bar{\phi}$, there is no default in the state L.**

Type 1

Substitute (A.2), (A.14) into (C.2), the change of type 1 consumption at date 0 is

$$dc_0^1\mid_{\phi_0=\bar{\phi_0}}= -\bar{\theta_{0b}}^1 dm_0^b - (\bar{\gamma_L}^1 + \bar{\gamma_H}^1 + \frac{\bar{\phi_0}\bar{\mu_0}^1}{\bar{\lambda_0}^1})d\theta_{0b}^1 -$$
$$- (\bar{\gamma_L}^1\bar{p_L} + \bar{\gamma_H}^1\bar{p_H} + \frac{\bar{\mu_0}^1}{\bar{\lambda_0}^1})dk_0.$$

Since both collateral constraint bind, $k_0 = -\phi_0\theta_{0b}^1$. Hence, the marginal changes $dk_0 = -\bar{\phi_0}d\theta_{0b}^1 - \bar{\theta_{0b}}^1 d\phi_0$, which yields,

$$dc_0^1\mid_{\phi_0=\bar{\phi_0}}= -\bar{\theta_{0b}}^1 dm_0^b - (\bar{\gamma_L}^1 + \bar{\gamma_H}^1 + \frac{\bar{\phi_0}\bar{\mu_0}^1}{\bar{\lambda_0}^1})d\theta_{0b}^1 -$$
$$(\bar{\gamma_L}^1\bar{p_L} + \bar{\gamma_H}^1\bar{p_H} + \frac{\bar{\mu_0}^1}{\bar{\lambda_0}^1})(-\bar{\phi_0}d\theta_{0b}^1 - \bar{\theta_{0b}}^1 d\phi_0)$$

After simplifying, I obtain

$$dc_0^1\mid_{\phi_0=\bar{\phi_0}}= -\bar{\theta_{0b}}^1 dm_0^b + \bar{\phi_0}\bar{\gamma_H}^1\bar{p_H}d\theta_{0b}^1 + \bar{\phi_0}\bar{\gamma_L}^1\bar{p_L}d\theta_{0b}^1 - \bar{\gamma_H}^1 d\theta_{0b}^1 - \bar{\gamma_L}^1 d\theta_{0b}^1 + (\bar{\gamma_L}^1\bar{p_L} + \bar{\gamma_H}^1\bar{p_H} + \frac{\bar{\mu_0}^1}{\bar{\lambda_0}^1})\bar{\theta_{0b}}^1 d\phi_0 \tag{C.36}$$

The change of type 1 consumption in the L state is

$$dc_L^1\mid_{\phi_0=\bar{\phi_0}}= -\bar{\theta_{Lb}}^1 dm_L^b - \bar{m_L^b}d\theta_{Lb}^1 - \bar{p_L}dk_L^1 - \bar{k_L}^1 dp_L + d\theta_{0b}^1 + \bar{p_L}dk_0 + \bar{k_0}dp_L. \tag{C.37}$$

Then, substitute (A.3), (A.4) into (C.37), which are from first order conditions for an optimum at the stationary competitive equilibrium:

$$\begin{aligned}
dc_L^1\mid_{\phi_0=\bar{\phi_0}}= & -\bar{\theta_{Lb}}^1 dm_L^b - \bar{m_L^b}d\theta_{Lb}^1 - \bar{p_L}dk_L^1 - \bar{k_L}^1 dp_L + d\theta_{0b}^1 + \bar{p_L}dk_0 + \bar{k_0}dp_L \\
= & -\bar{\theta_{Lb}}^1 dm_L^b - (\bar{\gamma_{LL}}^1\bar{\phi_L}r_{LL}^k + \bar{\gamma_{LH}}^1 + \frac{\bar{\phi_L}\bar{\mu_L}^1}{\bar{\lambda_L}^1})d\theta_{Lb}^1 - (\bar{\gamma_{LL}}^1 r_{LL}^k + \bar{\gamma_{LH}}^1 r_{LH}^k + \frac{\bar{\mu}_L^1}{\bar{\lambda_L}^1})dk_L^1 \\
& - \bar{k_L}^1 dp_L + d\theta_{0b}^1 + \bar{p_L}dk_0 + \bar{k_0}dp_L \\
= & -\bar{\theta_{Lb}}^1 dm_L^b - \bar{\gamma_{LH}}^1 d\theta_{Lb}^1 - \bar{\gamma_{LH}}^1 r_{LH}^k dk_L^1 - \bar{k_L}^1 dp_L + d\theta_{0b}^1 + \bar{p_L}dk_0 + \bar{k_0}dp_L
\end{aligned} \tag{C.38}$$

$dc_H^1\mid_{\phi_0=\bar{\phi_0}}$, $dc_{HH}^1\mid_{\phi_0=\bar{\phi_0}}$, $dc_{HL}^1\mid_{\phi_0=\bar{\phi_0}}$, $dc_{LH}^1\mid_{\phi_0=\bar{\phi_0}}$ and $dc_{LL}^1\mid_{\phi_0=\bar{\phi_0}}$ is not influenced by the default in the state L.

Thus, substitute equations (C.36), (C.38), (C.7), (C.8), (C.9), (C.10) and (C.11) into equation (C.1), and the marginal change of agent 1's utility is

$$\begin{aligned}
\frac{du^1}{\bar{\lambda_0}^1}\mid_{\phi_0=\bar{\phi_0}} = & -\bar{\theta_{0b}}^1 dm_0^b + \frac{\bar{\mu_0}^1}{\bar{\lambda_0}^1}\bar{\theta_{0b}}^1 d\phi_0 \\
& + \bar{\gamma_L}^1(-\bar{\theta_{Lb}}^1 dm_L^b + \bar{k_0}dp_L - \bar{k_L}^1 dp_L) + \bar{\gamma_H}^1(\bar{k_0}dp_H - \bar{k_H}^1 dp_H).
\end{aligned} \tag{C.39}$$

Type 2

I substitute (A.15) into (C.13), the change of type 2 consumption at date 0 is

$$dc_0^2\mid_{\phi_0=\bar{\phi_0}}= -\bar{\theta_{0b}}^2 dm^b - (\bar{\gamma_L}^2 + \bar{\gamma_H}^2)d\theta_{0b}^2. \tag{C.40}$$

The change of type 2 consumption in the L state is

$$dc_L^2 \mid_{\phi_0=\bar{\phi_0}} = -\bar{\theta}_{Lb}^2 dm_L^b - \bar{m}_L^b d\theta_{Lb}^2 - \bar{p}_L dk_L^2 - \bar{k}_L^2 dp_L + d\theta_{0b}^2. \tag{C.41}$$

Then, substitute (A.7) and (A.8) into (C.41), I obtain

$$dc_L^2 \mid_{\phi_0=\bar{\phi_0}} = -\bar{\theta}_{Lb}^2 dm_L^b - (\bar{\gamma}_{LL}^2 \bar{\phi}_L r_{LL}^k + \bar{\gamma}_{LH}^2) d\theta_{Lb}^2 - (\bar{\gamma}_{LL}^2 r_{LL}^k + \bar{\gamma}_{LH}^2 r_{LH}^k) dk_L^2 - \bar{k}_L^2 dp_L + d\theta_{0b}^2 \tag{C.42}$$

$dc_H^2 \mid_{\phi_0=\bar{\phi_0}}$, $dc_{HH}^2 \mid_{\phi_0=\bar{\phi_0}}$, $dc_{HL}^2 \mid_{\phi_0=\bar{\phi_0}}$, $dc_{LH}^2 \mid_{\phi_0=\bar{\phi_0}}$ and $dc_{LL}^2 \mid_{\phi_0=\bar{\phi_0}}$ is not influenced by the default in the state L.

Thus, substitute equations (C.40), (C.42), (C.18), (C.19), (C.20), (C.21) and (C.22) into equation (C.1), and the marginal change of agent 2's utility is

$$\begin{aligned} \frac{du^2}{\bar{\lambda}_0^2} \mid_{\phi_0=\bar{\phi_0}} &= -\bar{\theta}_{0b}^2 dm_0^b \\ &+ \bar{\gamma}_L^2(-\bar{\theta}_{Lb}^2 dm_L^b - \bar{k}_L^2 dp_L) + \bar{\gamma}_H^2(-\bar{k}_H^2 dp_H). \end{aligned} \tag{C.43}$$

Type 3

Substitute (A.16) into (C.24), I obtain, the change of type 3 consumption at date 0 is

$$dc_0^3 \mid_{\phi_0=\bar{\phi_0}} = -\bar{\theta}_{0b}^3 dm^b - (\bar{\gamma}_L^3 + \bar{\gamma}_H^3) d\theta_{0b}^2. \tag{C.44}$$

The change of type 3 consumption in the L state is

$$dc_L^3 \mid_{\phi_0=\bar{\phi_0}} = -\bar{\theta}_{Lb}^3 dm_L^b - \bar{m}_L^b d\theta_{Lb}^3 - \bar{p}_L dk_L^3 - \bar{k}_L^3 dp_L + d\theta_{0b}^3. \tag{C.45}$$

Then, substitute (A.12) and (A.13) into (C.45), I obtain

$$dc_L^3 \mid_{\phi_0=\bar{\phi_0}} = -\bar{\theta}_{Lb}^3 dm_L^b - (\bar{\gamma}_{LL}^3 \bar{\phi}_L r_{LL}^k + \bar{\gamma}_{LH}^3) d\theta_{Lb}^3 - (\bar{\gamma}_{LL}^3 r_{LL}^k + \bar{\gamma}_{LH}^3 r_{LH}^k) dk_L^3 - \bar{k}_L^3 dp_L + d\theta_{0b}^3 \tag{C.46}$$

$dc_H^3 \mid_{\phi_0=\bar{\phi_0}}$, $dc_{HH}^3 \mid_{\phi_0=\bar{\phi_0}}$, $dc_{HL}^3 \mid_{\phi_0=\bar{\phi_0}}$, $dc_{LH}^3 \mid_{\phi_0=\bar{\phi_0}}$ and $dc_{LL}^3 \mid_{\phi_0=\bar{\phi_0}}$ is not influenced by the default in the state L.

Thus, substitute equations (C.44), (C.46), (C.29), (C.30), (C.31), (C.32) and (C.33) into equation (C.1), and the marginal change of agent 3's utility is

$$\begin{aligned} \frac{du^3}{\bar{\lambda}_0^3} \mid_{\phi_0=\bar{\phi_0}} &= -\bar{\theta}_{0b}^3 dm_0^b \\ &+ \bar{\gamma}_L^3(-\bar{\theta}_{Lb}^3 dm_L^b - \bar{k}_L^3 dp_L) + \bar{\gamma}_H^3(-\bar{k}_H^3 dp_H). \end{aligned} \tag{C.47}$$

Social welfare

I add up equations (C.39), (C.43) and (C.47), and then

$$\sum_{i=1}^{3} w^i \frac{du^i}{\bar{\lambda_0}^i} |_{\phi_0=\bar{\phi_0}} = \frac{\bar{\mu_0}^1}{\bar{\lambda_0}^1}\bar{\theta_{0b}}^1 d\phi_0$$
$$+\sum_{i=1}^{3} w^i \bar{\gamma_L}^i(-\bar{\theta_{Lb}}^i dm_L^b)$$
$$+ w^1\bar{\gamma_H}^1\bar{k_0}dp_H + \sum_{i=1}^{3} w^i\bar{\gamma_H}^i(-\bar{k_H}^i dp_H)$$
$$+ w^1\bar{\gamma_L}^1\bar{k_0}dp_L + \sum_{i=1}^{3} w^i\bar{\gamma_L}^i(-\bar{k_L}^i dp_L). \tag{C.48}$$

Since $\sum_{i=1}^{3} w^i\bar{k_L}^i = \bar{k_0}$ and $\bar{\gamma_L}^1 < \bar{\gamma_L}^2 < \bar{\gamma_L}^3$, the fourth line of the equation (C.48) is

$$w^1\bar{\gamma_L}^1\bar{k_0}dp_L + \sum_{i=1}^{3} w^i\bar{\gamma_L}^i(-\bar{k_L}^i dp_L)$$
$$=(w^2(\bar{\gamma_L}^1 - \bar{\gamma_L}^2)\bar{k_L}^2 + w^3(\bar{\gamma_L}^1 - \bar{\gamma_L}^3)\bar{k_L}^3)dp_L.$$

Since $\bar{\gamma_L}^1 < \bar{\gamma_L}^2 < \bar{\gamma_L}^3$, the positive sign of the third line requires $dp_L < 0$.

II. Macro-prudential perturbations in the CPI-economy

In the CPI-economy, the perturbation dh_0 at the initial state, where $dh_0 > 0$ induce marginal changes at date 0, $(dc_0^i, dk_0, d\theta_{0b}^i, d\theta_{0j}^i)$,with $\sum_{i=1}^{3} w^i d\theta_{0b}^i = 0$, $\sum_{i=1}^{3} w^i d\theta_{0j}^i = 0$ and then adjustments of the subsequent equilibrium plans and prices $(dc_s^i, dk_L^i, dk_H^i, dm_0^b, dm_0^j, dm_L^b, dp_L, dp_H)$

around the equilibrium $(\bar{c_s}^i, \bar{k_0}, \bar{k_L}^i, \bar{k_H}^i, \bar{p_H}, \bar{p_L}, \bar{\theta_{0b}}^i, \bar{\theta_{0j}}^i, \bar{\theta_{Lb}}^i, \bar{m_0^b}, \bar{m_0^j}, \bar{m_L^b})$.

Then, I compute the marginal change of consumption distribution of each type of agents, relative to the stationary competitive equilibrium, following a marginal change of the policy parameter dh_0. Because of market clearing conditions, the effect on social welfare does not require compute dm_0^b and dm_0^j. Then, I compute the marginal changes of utilities of each type of agents and social welfare.

The change in agents i's marginal utility is given by

$$\frac{du^i}{\bar{\lambda_0}^i} |_{h_0=\bar{h_0}} = dc_0^i + \bar{\gamma_L}^i(dc_L^i + \bar{\gamma_{LL}}^i dc_{LL}^i + \bar{\gamma_{LH}}^i dc_{LH}^i) + \bar{\gamma_H}^i(dc_H^i + \bar{\gamma_{HL}}^i dc_{HL}^i + \bar{\gamma_{HH}}^i dc_{HH}^i). \tag{C.49}$$

Type 1

The change of type 1 consumption at date 0 is

$$dc_0^1 |_{h_0=\bar{h_0}} = -\bar{\theta_{0b}}^1 dm_0^b - \bar{m_0^b} d\theta_{0b}^1 - \bar{\theta_{0j}}^1 dm_0^j - \bar{m_0^j} d\theta_{0j}^1 - a dk_0. \tag{C.50}$$

Then, substitute (B.1), (B.2) and (B.3) into (C.50), which are from first order conditions for an optimum at the stationary competitive equilibrium:

$$dc_0^1 |_{h_0=\bar{h_0}} = -\bar{\theta_{0b}}^1 dm_0^b - \bar{\theta_{0j}}^1 dm_0^j - (\bar{\gamma_L}^1(\bar{\phi_0}\bar{p_L} + \bar{h_0}r_L^j) + \bar{\gamma_H}^1 + \frac{\bar{\phi_0}\bar{\mu}_0^1 + \bar{h_0}\bar{\eta_0}^1}{\bar{\lambda_0}^1})d\theta_{0b}^1 -$$
$$(\bar{\gamma_L}^1 r_L^j + \frac{\bar{\eta_0}^1}{\bar{\lambda_0}^1})d\theta_{0j}^1 - (\bar{\gamma_L}^1\bar{p_L} + \bar{\gamma_H}^1\bar{p_H} + \frac{\bar{\mu_0}^1}{\bar{\lambda_0}^1})dk_0.$$

Since both collateral constraint bind, $k_0 = -\phi_0\theta^1_{0b}$. $\theta^1_{0j} = -h_0\theta^1_{0b}$. Hence, the marginal changes $dk_0 = -\bar{\phi_0}d\theta^1_{0b}$ and $d\theta^1_{0j} = -(\bar{\theta_{0b}}^1 dh_0 + \bar{h_0}d\theta^1_{0b})$, which yields

$$dc^1_0 \mid_{h_0=\bar{h_0}} = -\bar{\theta_{0b}}^1 dm^b_0 - \bar{\theta_{0j}}^1 dm^j_0 - (\bar{\gamma_L}^1(\bar{\phi_0}\bar{p_L} + \bar{h_0}r^j_L) + \bar{\gamma_H}^1 + \frac{\bar{\phi_0}\bar{\mu}^1_0 + \bar{h_0}\bar{\eta_0}^1}{\bar{\lambda_0}^1})d\theta^1_{0b} +$$

$$(\bar{\gamma_L}^1 r^j_L + \frac{\bar{\eta_0}^1}{\bar{\lambda_0}^1})(\bar{\theta_{0b}}^1 dh_0 + \bar{h_0}d\theta^1_{0b}) + (\bar{\gamma_L}^1\bar{p_L} + \bar{\gamma_H}^1\bar{p_H} + \frac{\bar{\mu_0}^1}{\bar{\lambda_0}^1})\bar{\phi_0}d\theta^1_{0b} \tag{C.51}$$

After simplifying, I obtain

$$dc^1_0 \mid_{h_0=\bar{h_0}} = -\bar{\theta_{0b}}^1 dm^b_0 + \bar{\phi_0}\bar{\gamma_H}^1\bar{p_H}d\theta^1_{0b} - \bar{\gamma_H}^1 d\theta^1_{0b} + (\bar{\gamma_L}^1 r^j_L + \frac{\bar{\eta}^1_0}{\bar{\lambda_0}^1})\bar{\theta_{0b}}^1 dh_0$$

The change of type 1 consumption in the L state is

$$dc^1_L \mid_{h_0=\bar{h_0}} = -\bar{\theta_{Lb}}^1 dm^b_L - \bar{m}^b_L d\theta^1_{Lb} - \bar{p_L}dk^1_L - \bar{k_L}^1 dp_L + (\bar{\phi_0}\bar{p_L} + \bar{h_0}r^j_L)d\theta^1_{0b} + \bar{\phi_0}\bar{\theta_{0b}}^1(1-\bar{h_0})dp_L + .$$

$$\bar{\theta_{0b}}^1(1-\bar{\phi_0}\bar{p_L})dh_0 + \bar{p_L}dk_0 + \bar{k_0}dp_L - \bar{\phi_0}\bar{\theta_{0j}}^1 dp_L + (1-\bar{\phi_0}\bar{p_L})d\theta^1_{0j} \tag{C.52}$$

Since collateral constraints require $dk_0 = -\bar{\phi_0}d\theta^1_{0b}$ and $d\theta^1_{0j} = -(\bar{\theta_{0b}}^1 dh_0 + \bar{h_0}d\theta^1_{0b})$, I obtain

$$dc^1_L \mid_{h_0=\bar{h_0}} = -\bar{\theta_{Lb}}^1 dm^b_L - \bar{m}^b_L d\theta^1_{Lb} - \bar{p_L}dk^1_L - \bar{k_L}^1 dp_L \tag{C.53}$$

Then, substitute (B.4), (B.5) into (C.53), which are from first order conditions for an optimum at the stationary competitive equilibrium:

$$\begin{aligned} dc^1_L \mid_{h_0=\bar{h_0}} &= -\bar{\theta_{Lb}}^1 dm^b_L - \bar{m}^b_L d\theta^1_{Lb} - \bar{p_L}dk^1_L - \bar{k_L}^1 dp_L \\ &= -\bar{\theta_{Lb}}^1 dm^b_L - (\bar{\gamma_{LL}}^1\bar{\phi_L}r^k_{LL} + \bar{\gamma_{LH}}^1 + \frac{\bar{\phi_L}\bar{\mu_L}^1}{\bar{\lambda_L}^1})d\theta^1_{Lb} - (\bar{\gamma_{LL}}^1 r^k_{LL} + \bar{\gamma_{LH}}^1 r^k_{LH} + \frac{\bar{\mu}^1_L}{\bar{\lambda_L}^1})dk^1_L - \bar{k_L}^1 dp_L \\ &= -\bar{\theta_{Lb}}^1 dm^b_L - \bar{\gamma_{LH}}^1 d\theta^1_{Lb} - \bar{\gamma_{LH}}^1 r^k_{LH}dk^1_L - \bar{k_L}^1 dp_L \end{aligned} \tag{C.54}$$

The change of type 1 consumption in the H state is

$$dc^1_H \mid_{h_0=\bar{h_0}} = -\bar{p_H}dk^1_H - \bar{k_H}^1 dp_H + d\theta^1_{0b} + \bar{p_H}dk_0 + \bar{k_0}dp_H. \tag{C.55}$$

Then, substitute (B.6) into (C.55), which are from first order conditions for an optimum at the stationary competitive equilibrium:

$$dc^1_H \mid_{h_0=\bar{h_0}} = -\bar{k_H}^1 dp_H + d\theta^1_{0b} + (\bar{\gamma_{HL}}^1 r^k_{HL} + \bar{\gamma_{HH}}^1 r^k_{HH})(dk_0 - dk^1_H) + \bar{k_0}dp_H. \tag{C.56}$$

The change of type 1 consumption in the LH state is,

$$dc^1_{LH} \mid_{h_0=\bar{h_0}} = d\theta^1_{Lb} + r_{LH}dk^1_L. \tag{C.57}$$

The change of type 1 consumption in the LL state is,

$$dc^1_{LL} \mid_{h_0=\bar{h_0}} = \bar{\phi_L}r^k_{LL}d\theta^1_{Lb} + r_{LL}dk^1_L.$$

Since the collateral requirement at state L requires $dk^1_L = -\bar{\phi_L}d\theta^1_{Lb}$,

$$dc^1_{LL} \mid_{h_0=\bar{h_0}} = \bar{\phi_L}r^k_{LL}d\theta^1_{Lb} + r_{LL}(-\bar{\phi_L}d\theta^1_{Lb}) = 0. \tag{C.58}$$

The change of type 1 consumption in the HH state is,

$$dc^1_{HH} \mid_{h_0=\bar{h_0}} = r_{HH} dk^1_H. \tag{C.59}$$

The change of type 1 consumption in the HL state is,

$$dc^1_{HL} \mid_{\phi_0=\bar{\phi_0}} = r_{HL} dk^1_H. \tag{C.60}$$

Substitute equations (C.53), (C.55), (C.56),(C.57),(C.58),(C.59) and (C.60) into equation (C.49), and the marginal change of agent 1's utility is

$$\begin{aligned}\frac{du^1}{\bar{\lambda_0}^1} \mid_{h_0=\bar{h_0}} &= -\bar{\theta_{0b}}^1 dm^b_0 - \bar{\theta_{0j}}^1 dm^j_0 + (\bar{\gamma_L}^1 r^j_L + \frac{\bar{\eta_0}^1}{\bar{\lambda_0}^1})\bar{\theta_{0b}}^1 dh_0 \\ &+ \bar{\gamma_L}^1(-\bar{\theta_{Lb}}^1 dm^b_L - \bar{k_L}^1 dp_L) + \bar{\gamma_H}^1(\bar{k_0} dp_H - \bar{k_H}^1 dp_H).\end{aligned} \tag{C.61}$$

Type 2

The change of type 2 consumption at date 0 is

$$dc^2_0 \mid_{h_0=\bar{h_0}} = -\bar{\theta_{0b}}^2 dm^b_0 - \bar{m}^b_0 d\theta^2_{0b} - \bar{\theta_{0j}}^2 dm^j_0 - \bar{m}^j_0 d\theta^2_{0j} + \varepsilon d\theta^2_j. \tag{C.62}$$

Then, substitute (B.7) and (B.8) into (C.62), I obtain

$$dc^2_0 \mid_{h_0=\bar{h_0}} = -\bar{\theta_{0b}}^2 dm^b - (\bar{\gamma_L}^2(\bar{\phi_0}\bar{p_L} + \bar{h_0} r^j_L) + \bar{\gamma_H}^2) d\theta^2_{0b} - \bar{\theta_{0j}}^2 dm^j_0 - (\bar{\gamma_L}^2 r^j_L + \varepsilon) d\theta^2_{0j} + \varepsilon d\theta^2_j. \tag{C.63}$$

The change of type 2 consumption in the L state is

$$\begin{aligned}dc^2_L \mid_{h_0=\bar{h_0}} =& -\bar{\theta_{Lb}}^2 dm^b_L - \bar{m}^b_L d\theta^2_{Lb} - \bar{p_L} dk^2_L - \bar{k_L}^2 dp_L + (\bar{\phi_0}\bar{p_L} + \bar{h_0} r^j_L) d\theta^2_{0b} + \bar{\phi_0}\bar{\theta_{0b}}^2(1-\bar{h_0}) dp_L + \\ &\bar{\theta_{0b}}^2(1-\bar{\phi_0}\bar{p_L}) dh_0 - \bar{\phi_0}\bar{\theta_{0j}}^2 dp_L + r^j_L d\theta^2_{0j}\end{aligned} \tag{C.64}$$

Then, substitute (B.9) and (B.10) into (C.64), I obtain

$$\begin{aligned}dc^2_L \mid_{h_0=\bar{h_0}} =& -\bar{\theta_{Lb}}^2 dm^b_L - (\gamma\bar{_{LL}}^2\bar{\phi_L} r^k_{LL} + \gamma\bar{_{LH}}^2) d\theta^2_{Lb} - (\gamma\bar{_{LL}}^2 r^k_{LL} + \gamma\bar{_{LH}}^2 r^k_{LH}) dk^2_L - \bar{k_L}^2 dp_L \\ &+ \bar{\phi_0}\bar{\theta_{0b}}^2(1-\bar{h_0}) dp_L + \bar{\theta_{0b}}^2(1-\bar{\phi_0}\bar{p_L}) dh_0 - \bar{\phi_0}\bar{\theta_{0j}}^2 dp_L + r^j_L d\theta^2_{0j}\end{aligned} \tag{C.65}$$

The change of type 2 consumption in the H state is,

$$dc^2_H \mid_{h_0=\bar{h_0}} = -\bar{p_H} dk^2_H - \bar{k_H}^2 dp_H + d\theta^2_{0b}. \tag{C.66}$$

Then, substitute (B.11) into (C.66), I obtain

$$dc^2_H \mid_{h_0=\bar{h_0}} = -(\gamma\bar{_{HL}}^2 r^k_{HL} + \gamma\bar{_{HH}}^2 r^k_{HH}) dk^2_H - \bar{k_H}^2 dp_H + d\theta^2_{0b}. \tag{C.67}$$

The change of type 2 consumption in the LH state is,

$$dc^2_{LH} \mid_{h_0=\bar{h_0}} = d\theta^2_{Lb} + r_{LH} dk^2_L. \tag{C.68}$$

The change of type 2 consumption in the LL state is,

$$dc^2_{LL}\mid_{h_0=\bar{h_0}}=\bar{\phi_L} r^k_{LL} d\theta^2_{Lb}+r_{LL}dk^2_L. \tag{C.69}$$

The change of type 2 consumption in the HH state is,

$$dc^2_{HH}\mid_{h_0=\bar{h_0}}=r_{HH}dk^2_H. \tag{C.70}$$

The change of type 2 consumption in the HL state is,

$$dc^2_{HL}\mid_{h_0=\bar{h_0}}=r_{HL}dk^2_H. \tag{C.71}$$

Substitute equations (C.63), (C.65), (C.67), (C.68), (C.69), (C.70) and (C.71) into equation (C.49), and the marginal change of agent 2's utility is

$$\begin{aligned}\frac{du^2}{\bar{\lambda_0}^2}\mid_{h_0=\bar{h_0}} &= -\bar{\theta_{0b}}^2 dm^b_0-\bar{\theta_{0j}}^2 dm^j_0+\bar{\gamma_L}^2 r^j_L \bar{\theta_{0b}}^2 dh_0\\ &+\bar{\gamma_L}^2(-\bar{\theta_{Lb}}^2 dm^b_L-\bar{k_L}^2 dp_L-\bar{\phi_0}\bar{\theta_{0j}}^2 dp_L+\bar{\phi_0}\bar{\theta_{0b}}^2(1-\bar{h_0})dp_L)\\ &+\bar{\gamma_H}^2(-\bar{k_H}^2 dp_H).\end{aligned} \tag{C.72}$$

Type 3

The change of type 3 consumption at date 0 is

$$dc^3_0\mid_{h_0=\bar{h_0}}=-\bar{\theta_{0b}}^3 dm^b_0-\bar{m^b_0}d\theta^3_{0b}. \tag{C.73}$$

Then, substitute (B.12) into (C.60), I obtain

$$dc^3_0\mid_{h_0=\bar{h_0}}=-\bar{\theta_{0b}}^3 dm^b-(\bar{\gamma_L}^3(\bar{\phi_0}\bar{p_L}+\bar{h_0}r^j_L)+\bar{\gamma_H}^3)d\theta^2_{0b}. \tag{C.74}$$

The change of type 3 consumption in the L state is

$$\begin{aligned}dc^3_L\mid_{h_0=\bar{h_0}}=&-\bar{\theta_{Lb}}^3 dm^b_L-\bar{m^b_L}d\theta^3_{Lb}-\bar{p_L}dk^3_L-\bar{k_L}^3 dp_L+(\bar{\phi_0}\bar{p_L}+\bar{h_0}r^j_L)d\theta^3_{0b}+\\ &\bar{\phi_0}\bar{\theta_{0b}}^3(1-\bar{h_0})dp_L+\bar{\theta_{0b}}^3(1-\bar{\phi_0}\bar{p_L})dh_0\end{aligned} \tag{C.75}$$

Then, substitute (B.13) and (B.14) into (C.62), I obtain

$$\begin{aligned}dc^3_L\mid_{h_0=\bar{h_0}}=&-\bar{\theta_{Lb}}^3 dm^b_L-(\bar{\gamma_{LL}}^3\bar{\phi_L}r^k_{LL}+\bar{\gamma_{LH}}^3)d\theta^2_{Lb}-(\bar{\gamma_{LL}}^3 r^k_{LL}+\bar{\gamma_{LH}}^3 r^k_{LH})dk^3_L-\bar{k_L}^3 dp_L\\ &+(\bar{\phi_0}\bar{p_L}+\bar{h_0}r^j_L)d\theta^3_{0b}+\bar{\phi_0}\bar{\theta_{0b}}^3(1-\bar{h_0})dp_L+\bar{\theta_{0b}}^3(1-\bar{\phi_0}\bar{p_L})dh_0\end{aligned} \tag{C.76}$$

The change of type 3 consumption in the H state is,

$$dc^3_H\mid_{h_0=\bar{h_0}}=-\bar{p_H}dk^3_H-\bar{k_H}^3 dp_H+d\theta^3_{0b}. \tag{C.77}$$

Then, substitute (A.15) into (C.64), I obtain

$$dc^3_H\mid_{h_0=\bar{h_0}}=-(\bar{\gamma_{HL}}^3 r^k_{HL}+\bar{\gamma_{HH}}^3 r^k_{HH})dk^3_H-\bar{k_H}^3 dp_H+d\theta^3_{0b}. \tag{C.78}$$

The change of type 3 consumption in the LH state is,

$$dc^3_{LH}\mid_{h_0=\bar{h_0}}=d\theta^3_{Lb}+r_{LH}dk^3_L. \tag{C.79}$$

The change of type 3 consumption in the LL state is,

$$dc^3_{LL}\mid_{h_0=\bar{h_0}}=\bar{\phi_L}r^k_{LL}d\theta^3_{Lb}+r_{LL}dk^3_L. \tag{C.80}$$

The change of type 3 consumption in the HH state is,

$$dc^3_{HH}\mid_{h_0=\bar{h_0}}=r_{HH}dk^3_H. \tag{C.81}$$

The change of type 3 consumption in the HL state is,

$$dc^3_{HL}\mid_{h_0=\bar{h_0}}=r_{HL}dk^3_H. \tag{C.82}$$

Substitute equations (C.61), (C.63), (C.65), (C.66), (C.67), (C.68) and (C.69) into equation (C.36), and the marginal change of agent 3's utility is

$$\begin{aligned}\frac{du^3}{\bar{\lambda_0}^3}\mid_{h_0=\bar{h_0}} &= -\bar{\theta_{0b}}^3dm^b_0+\bar{\gamma_L}^3r^j_L\bar{\theta_{0b}}^3dh_0\\ &+\bar{\gamma_L}^3(-\bar{\theta_{Lb}}^3dm^b_L-\bar{k_L}^3dp_L+\bar{\phi_0}\bar{\theta_{0b}}^3(1-\bar{h_0})dp_L)\\ &+\bar{\gamma_H}^3(-\bar{k_H}^3dp_H).\end{aligned} \tag{C.83}$$

Social welfare

I add up equations (C.61), (C.72) and (C.83), and then

$$\begin{aligned}\sum_{i=1}^{3}w^i\frac{du^i}{\bar{\lambda_0}^i}\mid_{h=\bar{h_0}}=&(w^1\frac{du^1}{\bar{\lambda_0}^1}+w^2\frac{du^2}{\bar{\lambda_0}^2}+w^3\frac{du^3}{\bar{\lambda_0}^3})\mid_{h=\bar{h_0}}\\ =&w^1(-\bar{\theta_{0b}}^1dm^b_0-\bar{\theta_{0j}}^1dm^j_0+(\bar{\gamma_L}^1r^j_L+\frac{\bar{\eta_0}^1}{\bar{\lambda_0}^1})\bar{\theta_{0b}}^1dh_0\\ &+\bar{\gamma_L}^1(-\bar{\theta_{Lb}}^1dm^b_L-\bar{k_L}^1dp_L)+\bar{\gamma_H}^1(\bar{k_0}dp_H-\bar{k_H}^1dp_H))\\ &w^2(-\bar{\theta_{0b}}^2dm^b_0-\bar{\theta_{0j}}^2dm^j_0+\bar{\gamma_L}^2r^j_L\bar{\theta_{0b}}^2dh_0+\bar{\gamma_H}^2(-\bar{k_H}^2dp_H)\\ &+\bar{\gamma_L}^2(-\bar{\theta_{Lb}}^2dm^b_L-\bar{k_L}^2dp_L-\bar{\phi_0}\bar{\theta_{0j}}^2dp_L+\bar{\phi_0}\bar{\theta_{0b}}^2(1-\bar{h_0})dp_L))\\ &w^3(-\bar{\theta_{0b}}^3dm^b_0+\bar{\gamma_L}^3r^j_L\bar{\theta_{0b}}^3dh_0+\bar{\gamma_H}^3(-\bar{k_H}^3dp_H)\\ &+\bar{\gamma_L}^3(-\bar{\theta_{Lb}}^3dm^b_L-\bar{k_L}^3dp_L+\bar{\phi_0}\bar{\theta_{0b}}^3(1-\bar{h_0})dp_L))\\ =&-(w^1\bar{\theta_{0b}}^1+w^2\bar{\theta_{0b}}^2+w^3\bar{\theta_{0b}}^3)dm^b_0-(w^1\bar{\theta_{0j}}^1+w^2\bar{\theta_{0j}}^2)dm^j_0\\ &+w^1(\bar{\gamma_L}^1r^j_L+\frac{\bar{\eta_0}^1}{\bar{\lambda_0}^1})\bar{\theta_{0b}}^1dh_0+w^2\bar{\gamma_L}^2r^j_L\bar{\theta_{0b}}^2dh_0+w^3\bar{\gamma_L}^3r^j_L\bar{\theta_{0b}}^3dh_0\\ &-(w^1\bar{\gamma_L}^1\bar{\theta_{Lb}}^1+w^2\bar{\gamma_L}^2\bar{\theta_{Lb}}^2+w^3\bar{\gamma_L}^3\bar{\theta_{Lb}}^3)dm^b_L\\ &+(w^2\bar{\gamma_L}^2\bar{\phi_0}\bar{\theta_{0b}}^2(1-\bar{h_0})+w^3\bar{\gamma_L}^3\bar{\phi_0}\bar{\theta_{0b}}^3(1-\bar{h_0})-w^1\bar{\gamma_L}^1\bar{k_L}^1-w^2\bar{\gamma_L}^2\bar{k_L}^2-w^3\bar{\gamma_L}^3\bar{k_L}^3)dp_L\\ &+w^1\bar{\gamma_H}^1\bar{k_0}dp_H-(w^1\bar{\gamma_H}^1\bar{k_H}^1+w^2\bar{\gamma_H}^2\bar{k_H}^2+w^3\bar{\gamma_H}^3\bar{k_H}^3)dp_H\end{aligned}$$

When the contract markets are clear, $-w^1\bar{\theta_{sb}}^1=w^2\bar{\theta_{sb}}^2+w^3\bar{\theta_{sb}}^3$, $-w^1\bar{\theta_{0j}}^1=w^2\bar{\theta_{0j}}^2$. Since $\bar{m^j_0}=\bar{\gamma_L}^1r^j_L+\frac{\bar{\eta}^1}{\bar{\lambda_0}^1}=\bar{\gamma_L}^2r^j_L+\varepsilon$ and I add $w^3\bar{\gamma_L}^2r^j_L\bar{\theta_{0b}}^3dh_0$ and subtract it, then I obtain

$$\sum_{i=1}^{3} w^i \frac{du^i}{\bar{\lambda_0}^i} |_{h_0=\bar{h_0}} = w^3 r_L^j \bar{\theta_{0b}}^3 (\bar{\gamma_L}^3 - \bar{\gamma_L}^2) dh_0 + \varepsilon w^1 \bar{\theta_b}^1 dh_0$$
$$+ \sum_{i=1}^{3} w^i \bar{\gamma_L}^i (-\bar{\theta_{Lb}}^i dm_L^b)$$
$$+ w^1 \bar{\gamma_H}^1 \bar{k_0} dp_H + \sum_{i=1}^{3} w^i \bar{\gamma_H}^i (-\bar{k_H}^i dp_H)$$
$$+ ((w^2 \bar{\gamma_L}^2 \bar{\theta_{0b}}^2 + w^3 \bar{\gamma_L}^3 \bar{\theta_{0b}}^3 + \bar{h_0} \bar{\theta_{0b}}^3 (\bar{\gamma_L}^3 - \bar{\gamma_L}^2)) \bar{\phi_0} - \sum_{i=1}^{3} w^i \bar{\gamma_L}^i \bar{k_L}^i) dp_L. \quad \text{(C.84)}$$

If $\varepsilon < -w^3 \bar{\theta_{0b}}^3 r_L^j (\bar{\gamma_L}^3 - \bar{\gamma_L}^2)/w^1 \bar{\theta_{0b}}^1$, the first line of equation (C.84) is positive. The positive signs of the second and third lines in the equation (C.84) require the same condition in the equation (C.35). However, since $\bar{h_0} \bar{\theta_{0b}}^3 (\bar{\gamma_L}^3 - \bar{\gamma_L}^2) \bar{\phi_0} > 0$, the positive sign of the fourth line does not require $\bar{k_L}^1 > 0$.

References

Acharya, V., Engle, R., & Pierret, D. (2014). Testing macroprudential stress tests: The risk of regulatory risk weights. *Journal of Monetary Economics*, *65*, 36–53.

Araujo, A., Kubler, F., & Schommer, S. (2012). Regulating collateral-requirements when markets are incomplete. *Journal of Economic Theory*, *147*(2), 450–476.

Bengtsson, E. (2019). Macroprudential policy in the eu: A political economy perspective. *Global Finance Journal*, 100490.

Benigno, G., Chen, H., Otrok, C., Rebucci, A., & Young, E. R. (2013). Financial crises and macroprudential policies. *Journal of International Economics*, *89*(2), 453–470.

Bianchi, J., & Mendoza, E. G. (2018). Optimal time-consistent macroprudential policy. *Journal of Political Economy*, *126*(2), 588–634.

Boz, E., & Mendoza, E. G. (2014). Financial innovation, the discovery of risk, and the us credit crisis. *Journal of Monetary Economics*, *62*, 1–22.

Branch, W. A., & McGough, B. (2009). A new keynesian model with heterogeneous expectations. *Journal of Economic Dynamics and Control*, *33*(5), 1036–1051.

Brunnermeier, M. K. (2014, November). A welfare criterion for models with distorted beliefs*. *Quarterly Journal of Economics*, *129*(4), 1753–1798.

Brunnermeier, M. K., & Pedersen, L. H. (2009). Market liquidity and funding liquidity. *The review of financial studies*, *22*(6), 2201–2238.

Brunnermeier, M. K., & Sannikov, Y. (2014). A macroeconomic model with a financial sector. *American Economic Review*, *104*(2), 379–421.

Calem, P., Correa, R., & Lee, S. J. (2020). Prudential policies and their impact on credit in the united states. *Journal of Financial Intermediation*, *42*, 100826.

Claessens, S. (2015). An overview of macroprudential policy tools. *Annual Review of Financial Economics*, *7*, 397–422.

De Grauwe, P. (2011). Animal spirits and monetary policy. *Economic theory*, *47*(2-3), 423–457.

Dogra, K., & Gorbachev, O. (2016). Consumption volatility, liquidity constraints and household welfare. *The Economic Journal*, *126*(597), 2012–2037.

Dynan, K. E., Elmendorf, D. W., & Sichel, D. E. (2006). Can financial innovation help to explain the reduced volatility of economic activity? *Journal of monetary Economics*, *53*(1), 123–150.

Fostel, A., & Geanakoplos, J. (2012). Tranching, cds, and asset prices: How financial innovation can cause bubbles and crashes. *American Economic Journal: Macroeconomics*, *4*(1), 190–225.

Frankel, J. A., & Froot, K. A. (1985). *Using survey data to test some standard propositions regarding exchange rate expectations* (Tech. Rep.). National Bureau of Economic Research.

Geanakoplos, J. (1997). Promises, promises. *The economy as an evolving complex system II*, *27*, 285.

Geanakoplos, J. (2003). Liquidity, default, and crashes endogenous contracts in general. In *Advances in economics and econometrics: theory and applications: eighth world congress* (Vol. 170).

Geanakoplos, J. (2010a). The leverage cycle. *NBER macroeconomics annual*, *24*(1), 1–66.

Geanakoplos, J. (2010b). Solving the present crisis and managing the leverage cycle.

Geanakoplos, J., & Zame, W. R. (2014). Collateral equilibrium, i: a basic framework. *Economic Theory*, *56*(3), 443–492.

Kandel, E., & Pearson, N. D. (1995). Differential interpretation of public signals and trade in speculative markets. *Journal of Political Economy*, *103*(4), 831–872.

Kim, S., Plosser, M. C., & Santos, J. A. (2018). Macroprudential policy and the revolving door of risk: Lessons from leveraged lending guidance. *Journal of Financial Intermediation*, *34*, 17–31.

Kiyotaki, N., & Moore, J. (1997). Credit cycles. *Journal of political economy*, *105*(2), 211–248.

Korinek, A., & Simsek, A. (2016). Liquidity trap and excessive leverage. *American Economic Review*, *106*(3), 699–738.

Kurz, M., Jin, H., & Motolese, M. (2005). The role of expectations in economic fluctuations and the efficacy of monetary policy. *Journal of Economic Dynamics and Control*, *29*(11), 2017–2065.

Lopez, P., et al. (2016). Welfare implications of the term structure of returns: Should central banks fill gaps or remove volatility? In *2016 meeting papers.*

Morrison, A. D. (2005). Credit derivatives, disintermediation, and investment decisions. *The Journal of Business*, *78*(2), 621–648.

Rubio, M., & Carrasco-Gallego, J. A. (2014). Macroprudential and monetary policies: Implications for financial stability and welfare. *Journal of Banking & Finance*, *49*, 326–336.

Simsek, A. (2013). Belief disagreements and collateral constraints. *Econometrica*, *81*(1), 1–53.

Vousinas, G. L. (2015). Supervision of financial institutions: The transition from basel i to basel iii. a critical appraisal of the newly established regulatory framework. *Journal of Financial Regulation and Compliance.*

Wei, L. (2020). Financial innovation, collateral hedging and macro-prudential policies.